HETERARCHY

… a multi-scale journey

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SCIGEM
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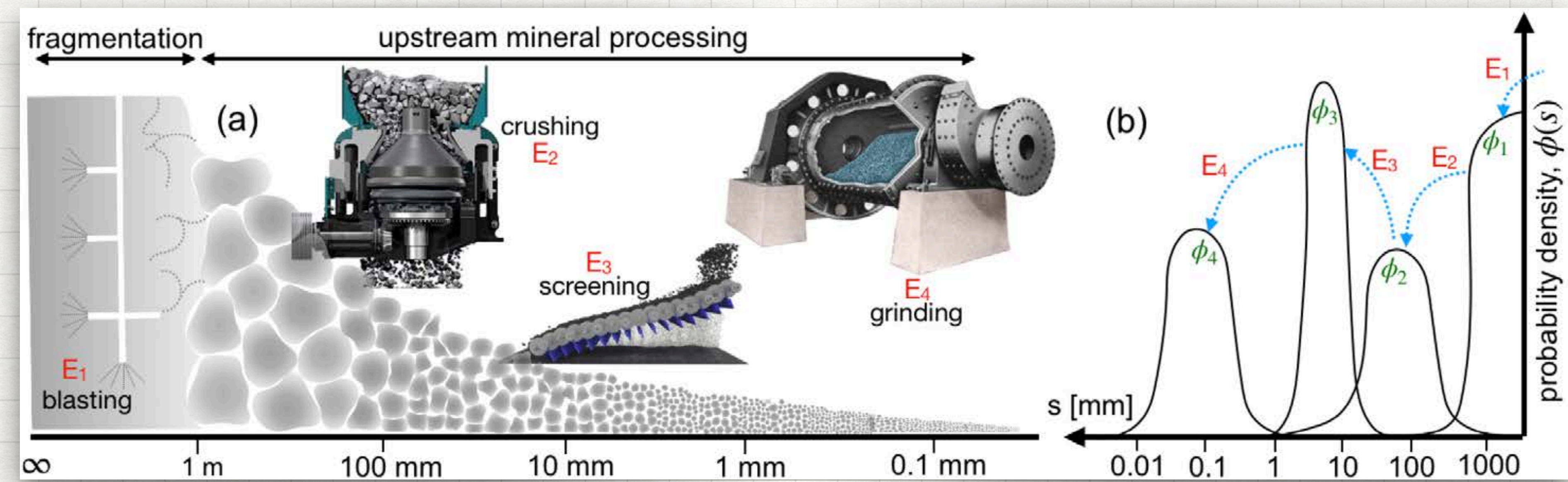
AUSTRALIA

MOTIVATION

- MINERAL PROCESSING

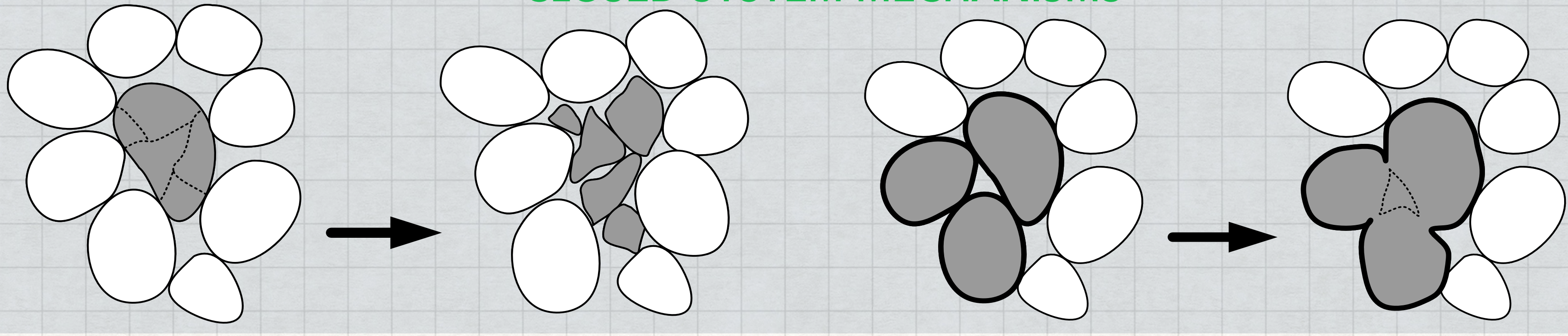


grainsize probability density function (PDD): $\phi \equiv \phi(\mathbf{x}, t, s)$

position
time
grainsize

PROCESSES

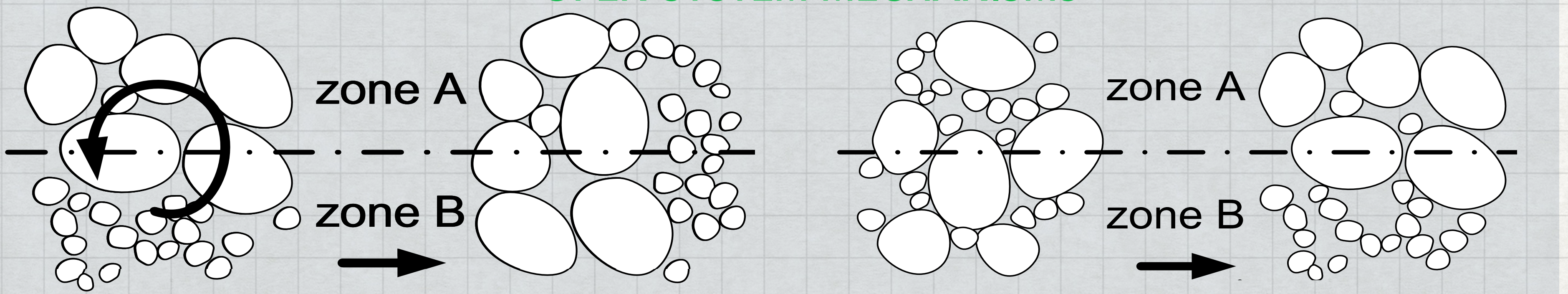
CLOSED-SYSTEM MECHANISMS



crushing

agglomeration

OPEN-SYSTEM MECHANISMS



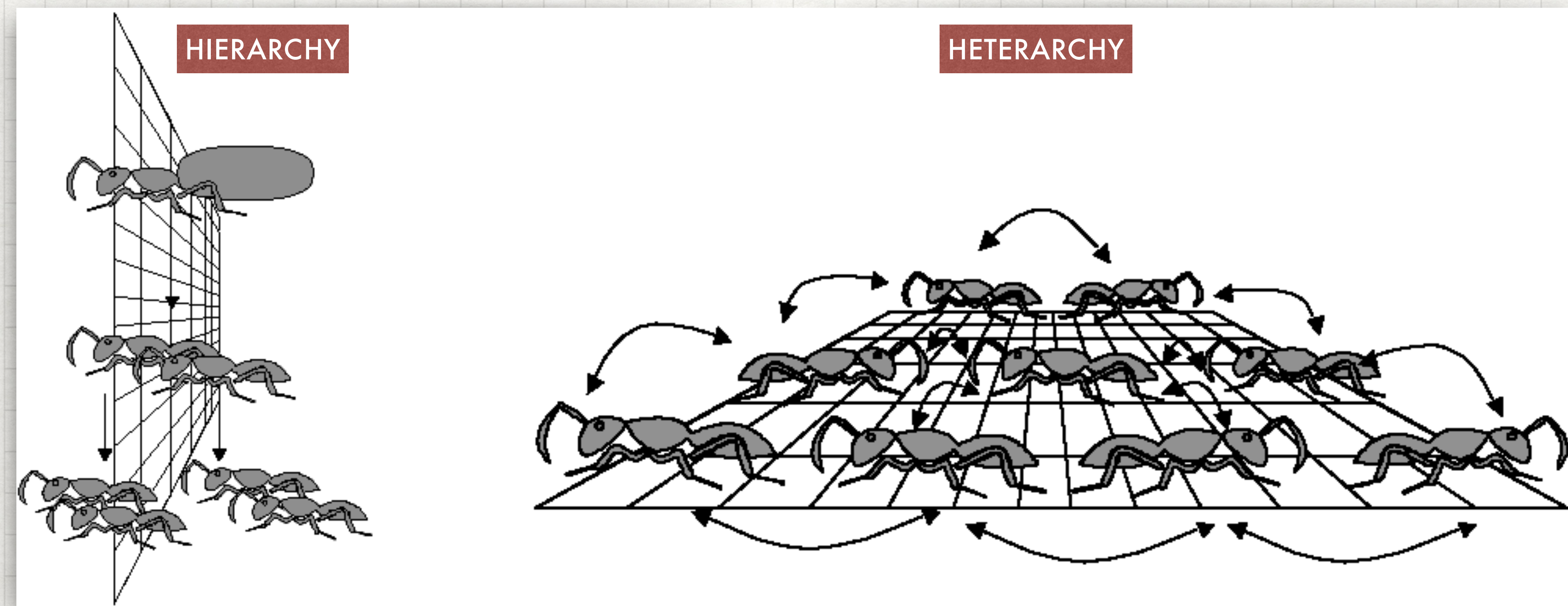
mixing

segregation

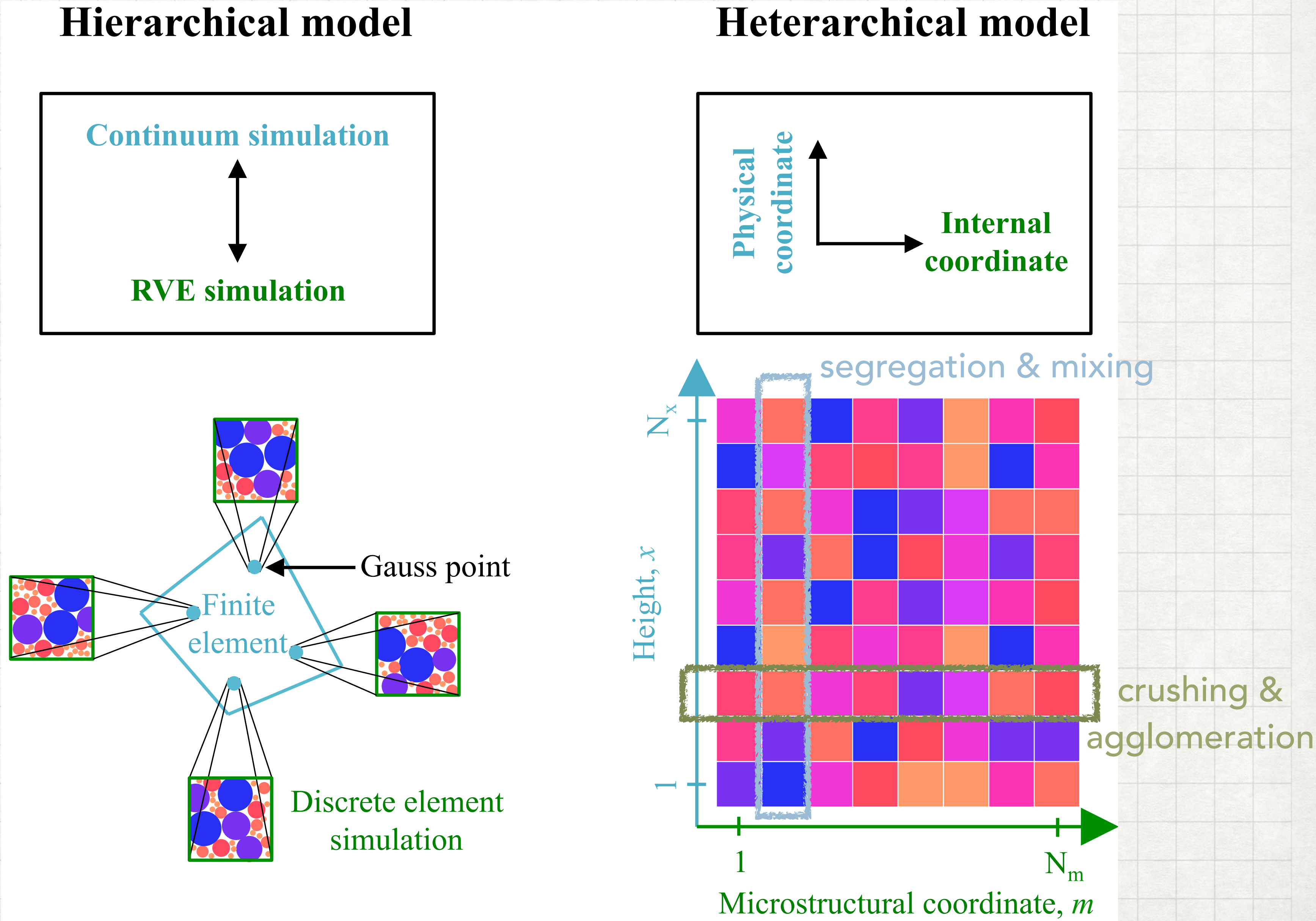
HETERARCHY

- IN ENTOMOLOGY AND BUSINESS

A HETERARCHY IS AN ORGANISATIONAL SYSTEM WITH UNRANKED ELEMENTS



MULTISCALE MODELS



Marks-Einav, GM, 2017

segregation

(from stochastic to continuum mechanics)

SEGREGATION

- ## PREVIOUS MODELS

J. Fluid Mech. (1988), *vol.* 189, *pp.* 311–335 311
Printed in Great Britain

Particle size segregation in inclined chute flow of dry cohesionless granular solids

By S. B. SAVAGE AND C. K. K. LUN

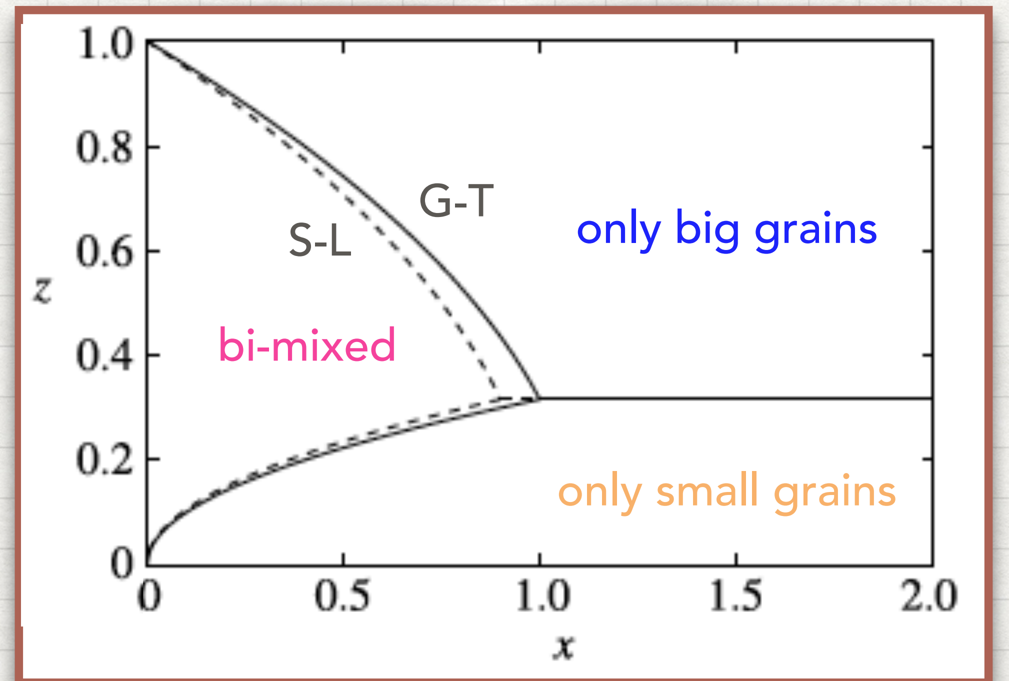
OR

PROCEEDINGS OF THE ROYAL SOCIETY A
Proc. R. Soc. A (2005) **461**, 1447–1473
doi:10.1098/rspa.2004.1420
Published online 26 April 2005

A theory for particle size segregation in shallow granular free-surface flows

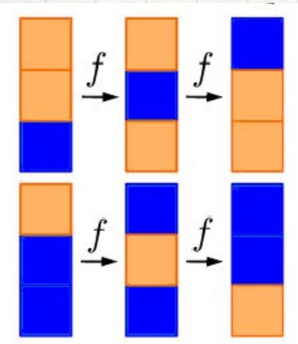
By J. M. N. T. Gray and A. R. Thornton

?

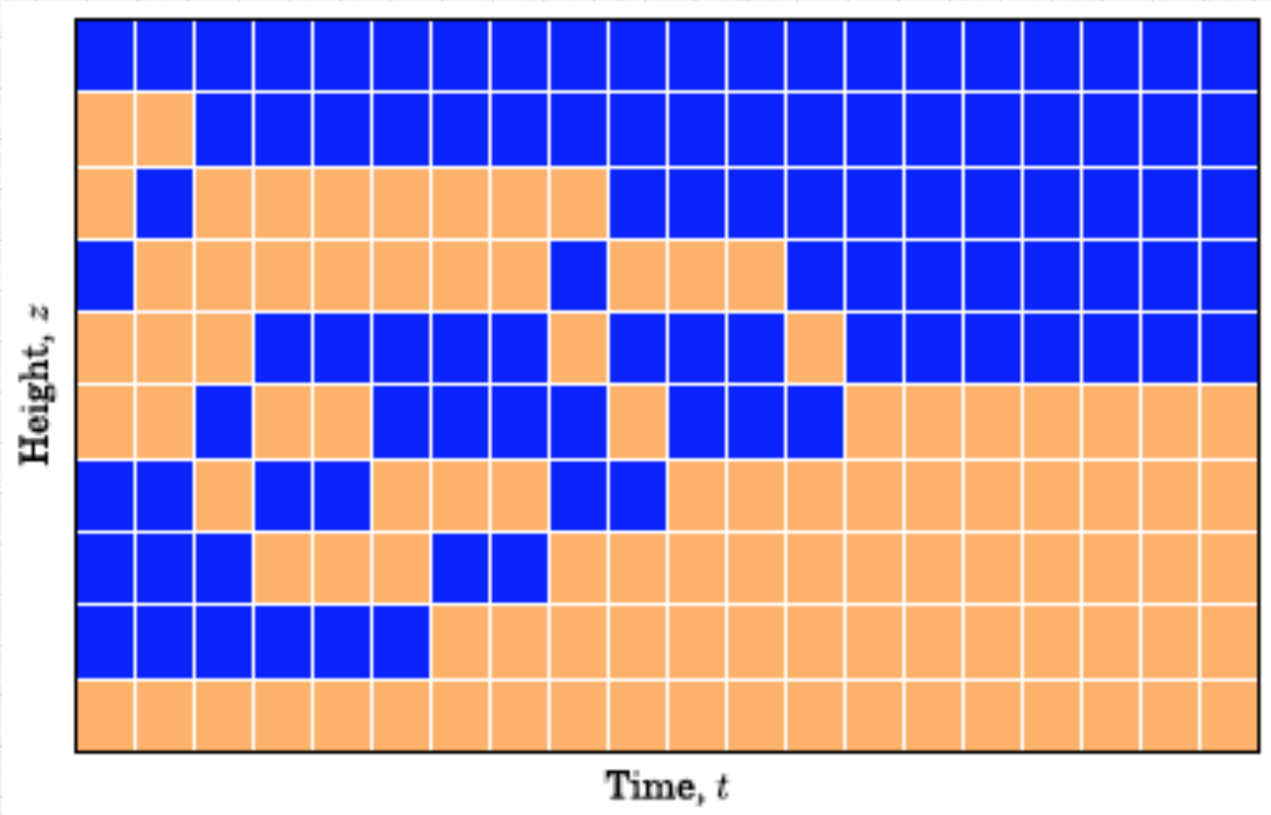


STOCHASTIC SEGREGATION

- STOCHASTIC LATTICE MODEL



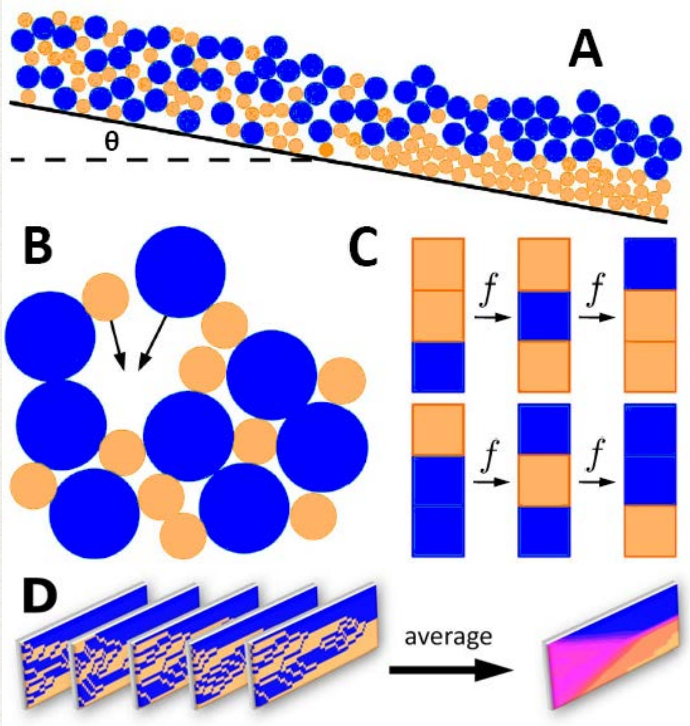
$$z(s, t + \Delta t) = z(s, t) + \hat{u}(z, s, t)\Delta t$$
$$= z(s, t) + f(z, s, t)\frac{\Delta t}{\Delta z}$$

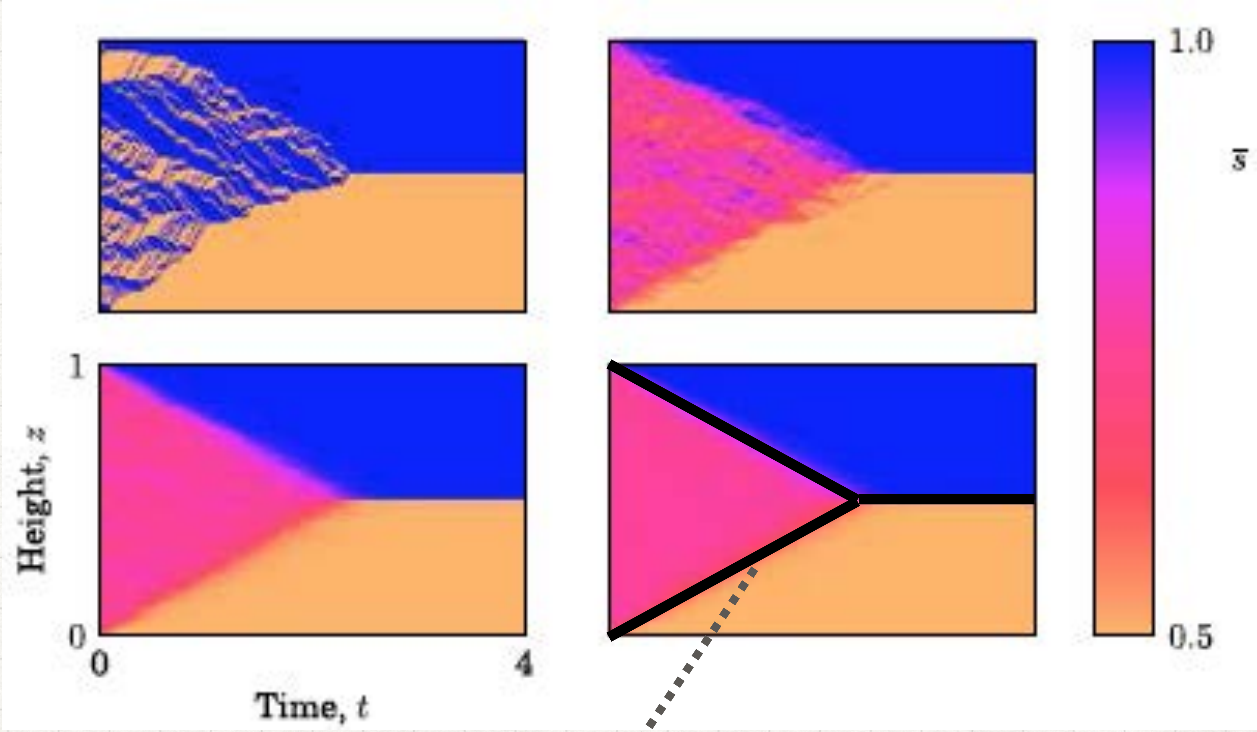


Marks-Einav, GM, 2011

SEGREGATION

- FROM STOCHASTIC TO CONTINUUM





$$\partial_t \Phi_s = C \partial_s \left[\Phi_s (1 - \Phi_s) \right]$$

$\hat{u} = C$ for now taken constant

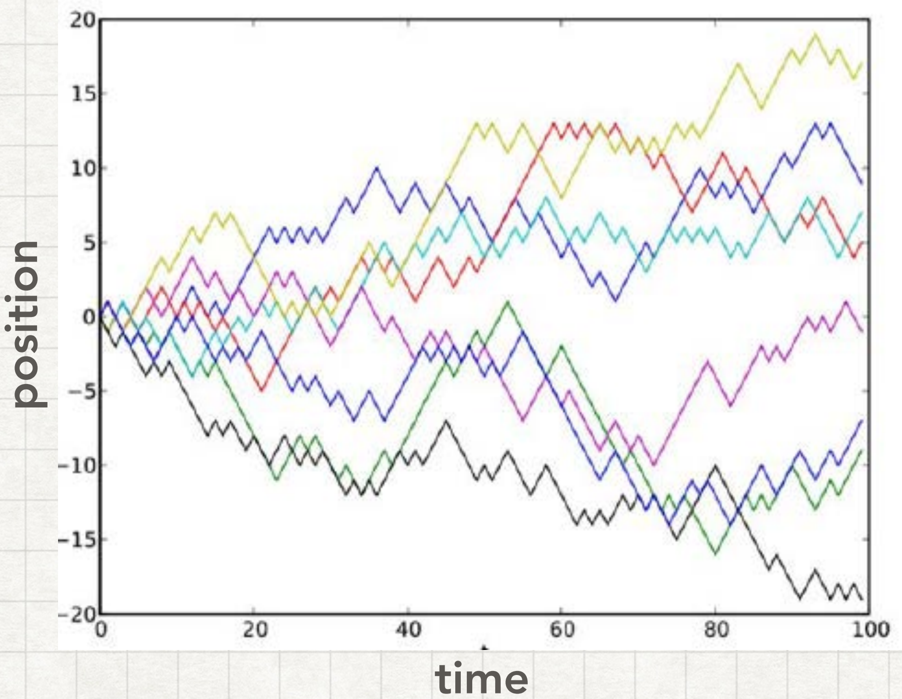
Marks-Einav, GM, 2011

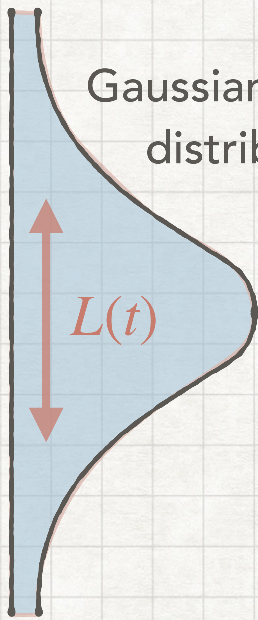
mixing

(from stochastic to continuum mechanics)

STOCHASTIC MIXING

- BROWNIAN MOTION (EINSTEIN, 1905)





Gaussian/Normal distribution

1D	β=2
2D	β=4
3D	β=6

Averaged position $\langle \mathbf{x_p(t)} - \mathbf{x_p(0)} \rangle \approx \mathbf{0}$

Diffusion length $L(t) = \sqrt{\langle \left(\mathbf{x_p(t)} - \mathbf{x_p(0)} \right)^2 \rangle} = \sqrt{\beta D t}$

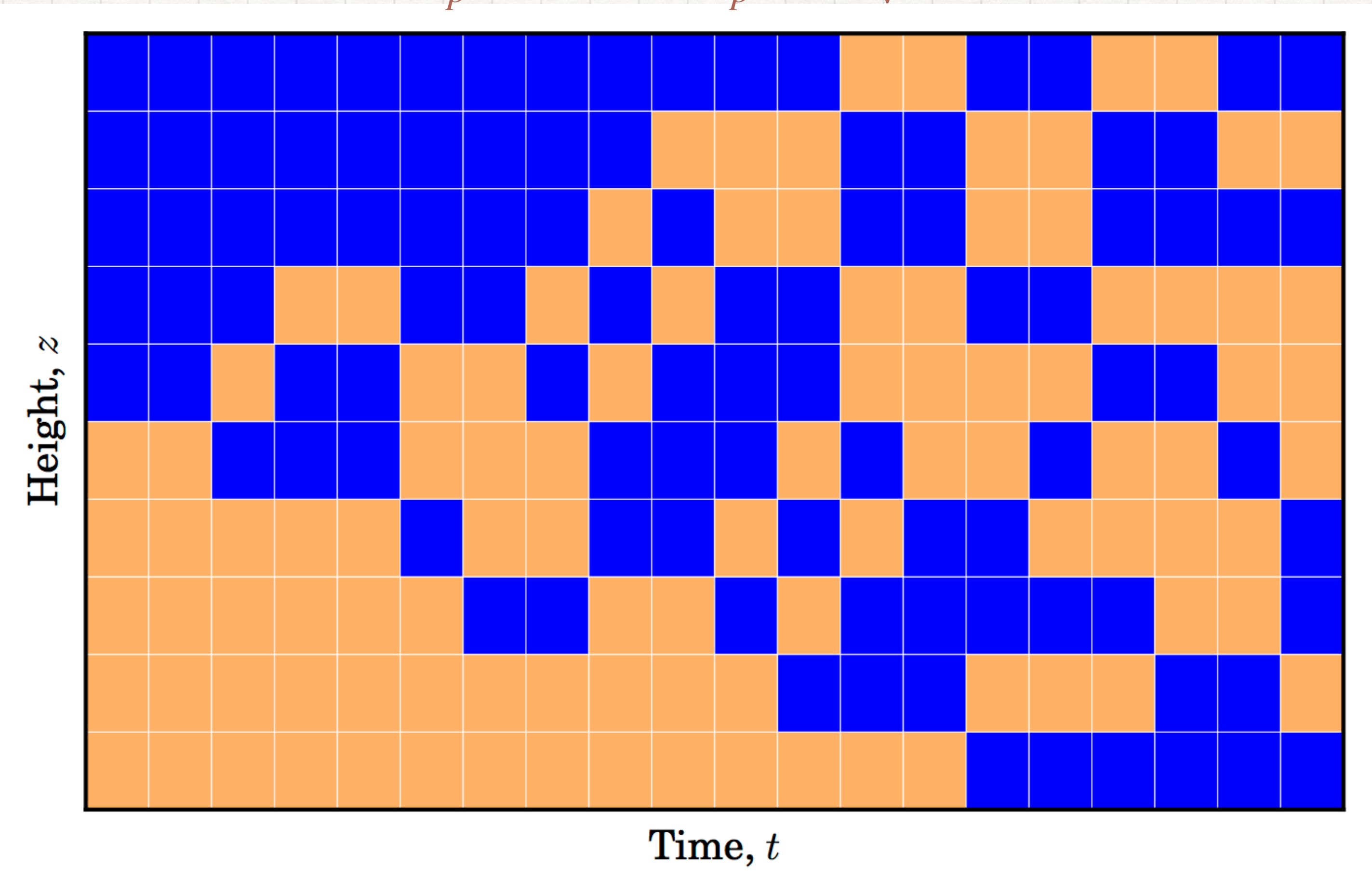
Diffusivity in m^2/s

STOCHASTIC MIXING

- ## STOCHASTIC LATTICE MODEL

$\beta = 2$

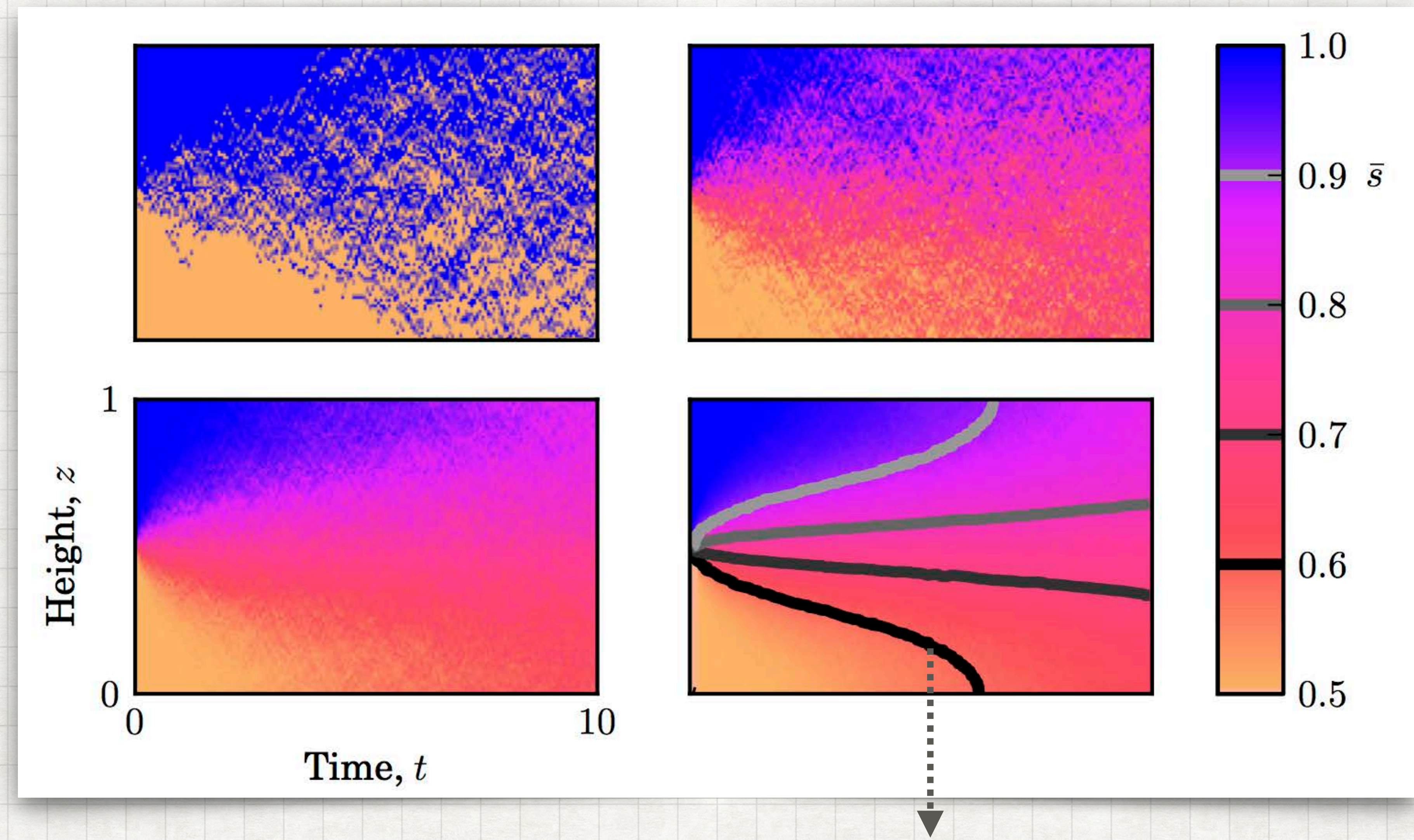
$$x_p(t + \Delta t) = x_p(t) \pm \sqrt{2D\Delta t}$$



Swap cells at a (stochastic) frequency determined by 1D diffusivity *D*

MIXING

- FROM STOCHASTIC TO CONTINUUM



Marks-Einav, GM, 2017

$$\partial_t \phi(\mathbf{x}, s, t) = D\nabla^2 \phi$$

grain crushing

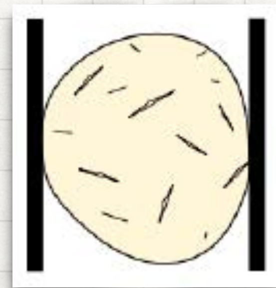
(from stochastic to continuum mechanics)

STOCHASTIC COMMINUTION

- FACTOR #1

critical crushing stress

$\sigma_{cr} = ?$



critical crushing stress

$\sigma_{cr} = ?$

$$\sigma_{pc} \propto d_p^{-3/w}$$

Mogi (1962);
McDowell-Bolton (1996)

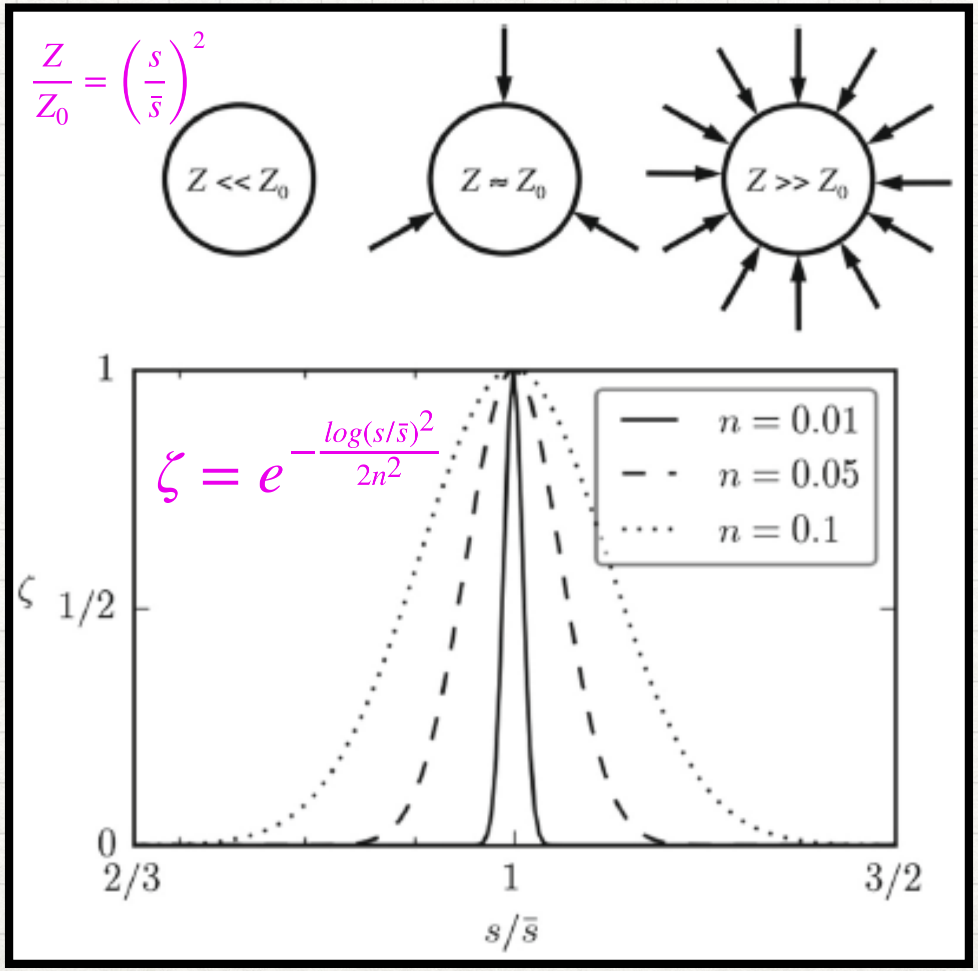
STOCHASTIC COMMINUTION

- FACTOR #2

crushing stress

bulk stress

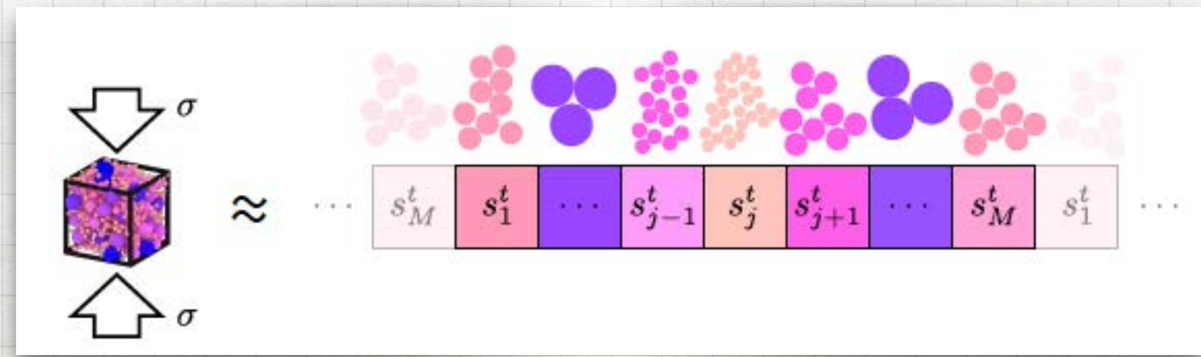
$$\sigma_{cr}(s, \bar{s}) = \zeta\,\sigma$$



Marks & Einav, GRL, 2017

STOCHASTIC COMMINUTION

- CRUSHING LAW



NEIGHBOURING GRAINSIZE FABRIC

Crushing criterion
$$s_j^{t+1} = \begin{cases} X_j^t s_j^t & \text{if } \sigma \geq \sigma_j^t\left(s_{j-1}, s_j, s_{j+1}\right) = \left(\dfrac{s_j}{s_{\max}}\right)^{-3/w} \exp\left(\dfrac{\log\left(s_j^t/\bar{s}_j^t\right)^2}{2n^2}\right) \\ s_j^t & \text{otherwise,} \end{cases}$$

#1 Weibull size dependence

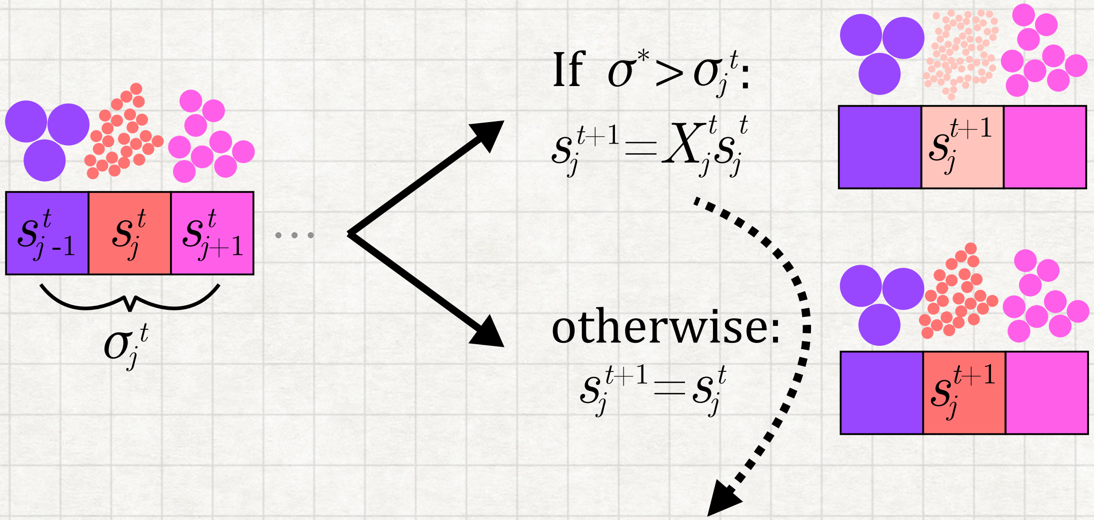
#2 cushioning

pull from fragment distribution

Marks & Einav, GRL, 2017

STOCHASTIC COMMINUTION

- FRAGMENTATION LAW



$$f(x) = \frac{e^{-x}}{1 - e^{-1}}$$

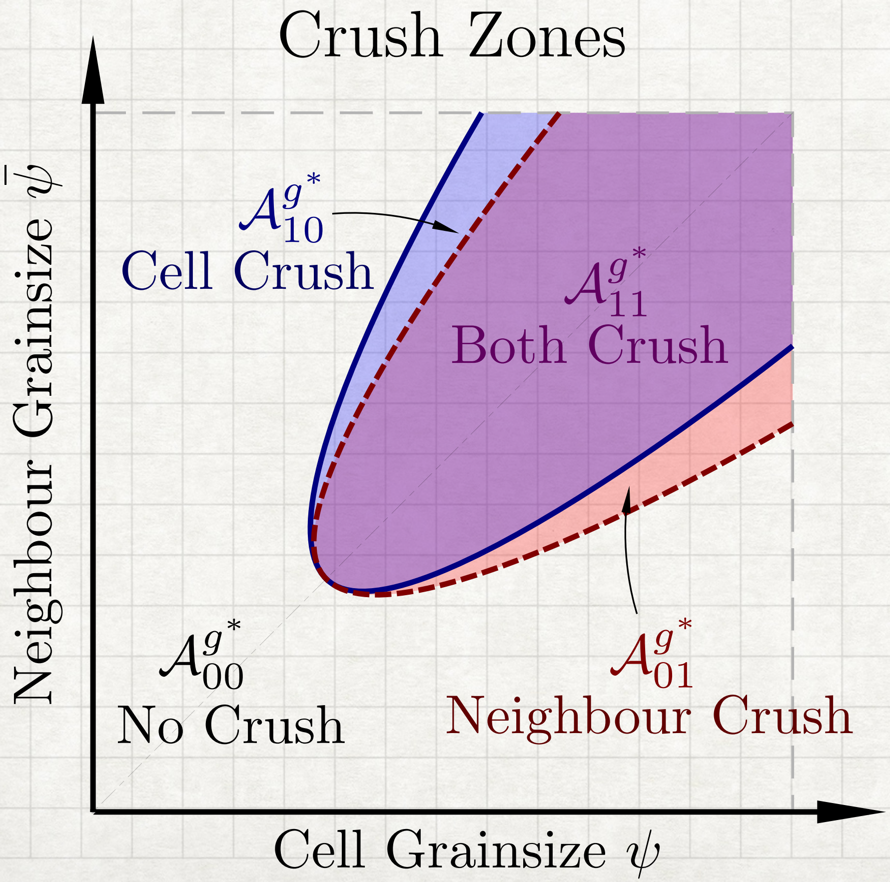
fragment size distribution

Marks & Einav, GRL, 2017

CONTINUUM HOMOGENISATION

TIME EVOLUTION

$$\rho^{t+1}(\psi,\bar{\psi}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d\phi\, d\bar{\phi}\, \rho^{t}(\phi,\bar{\phi})\, \kappa^{g^*}(\phi-\psi,\bar{\phi}-\bar{\psi})$$

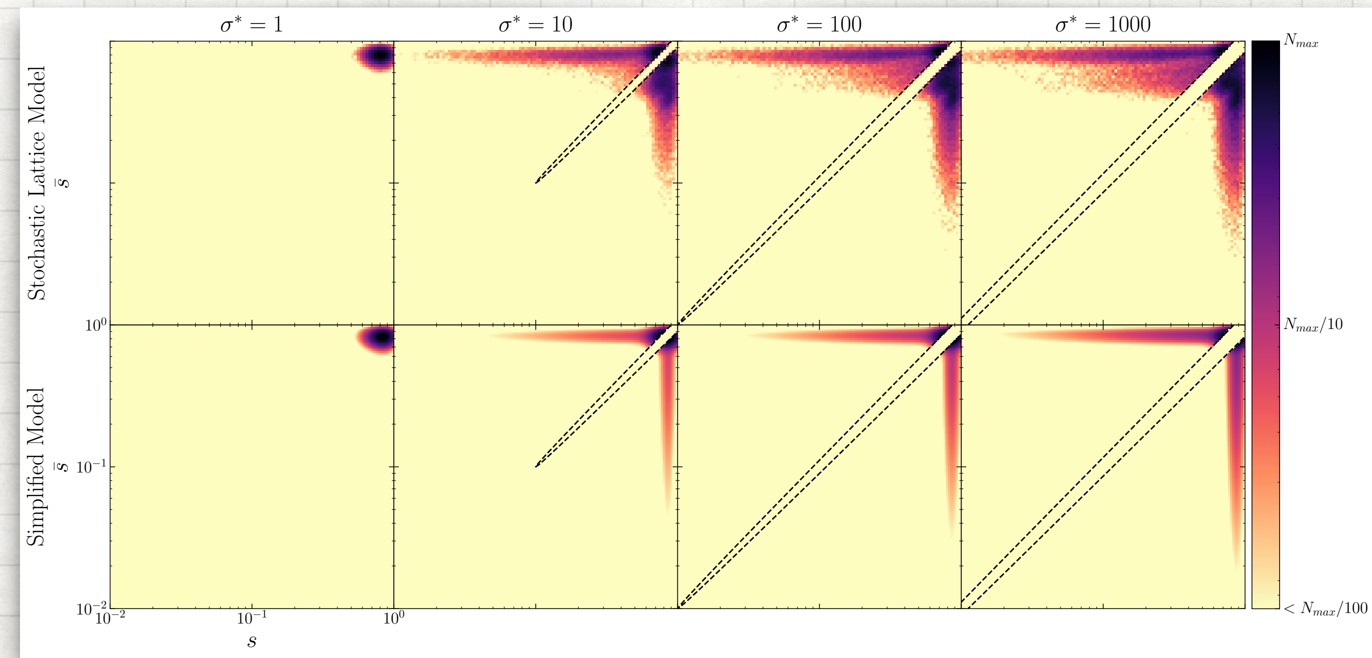


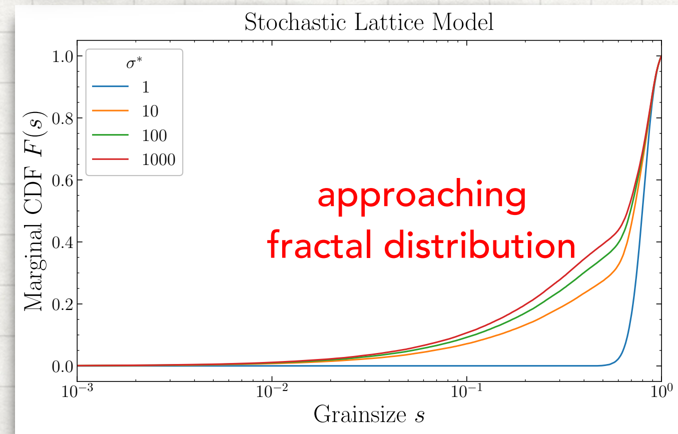
$$\kappa^{g^*}(\psi,\bar{\psi}) := \begin{cases} \delta(\psi)\delta(\bar{\psi}) & \text{if } (\psi,\bar{\psi}) \in \mathcal{A}_{00}^{g^*} \\ \delta(\psi)f(\bar{\psi}) & \text{if } (\psi,\bar{\psi}) \in \mathcal{A}_{01}^{g^*} \\ f(\psi)\delta(\bar{\psi}) & \text{if } (\psi,\bar{\psi}) \in \mathcal{A}_{10}^{g^*} \\ f(\psi)f(\bar{\psi}) & \text{if } (\psi,\bar{\psi}) \in \mathcal{A}_{11}^{g^*} \end{cases}$$

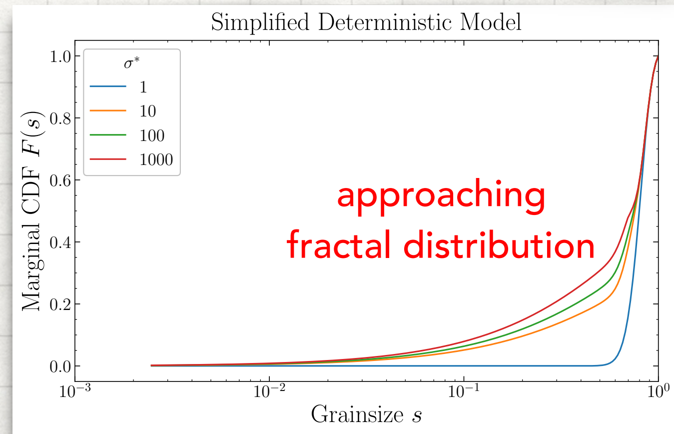
Huang-Marks-Einav, JMPS, 2020

CONTINUUM HOMOGENISATION

RESULTS (INITIALLY MONO-DISPERSED)





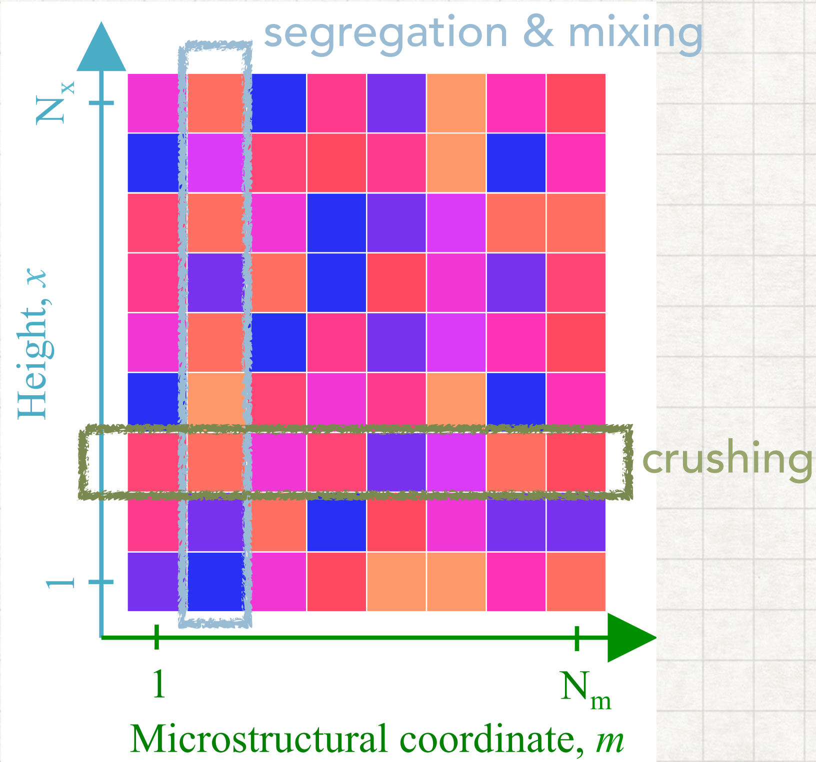


examples

(full heterarchy)

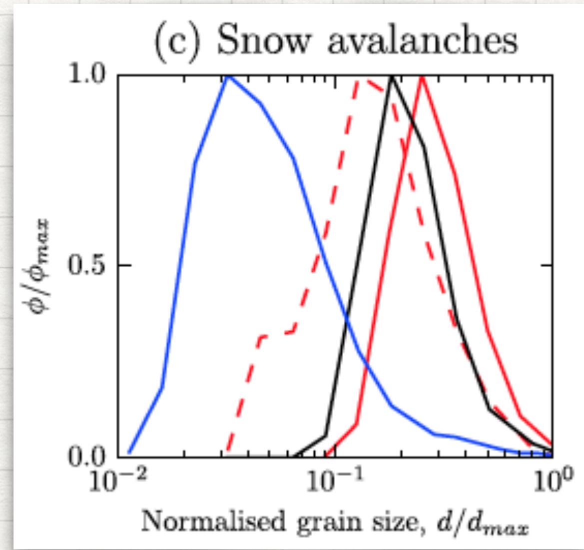
FULL HETERARCHY

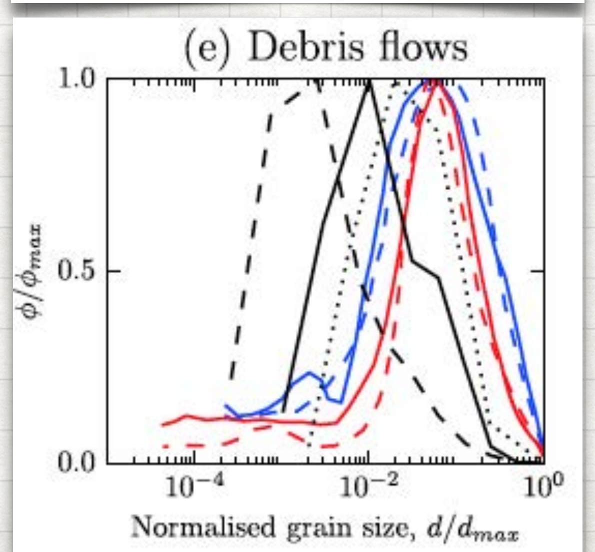
- SIMPLEST STOCHASTIC LATTICE MODEL

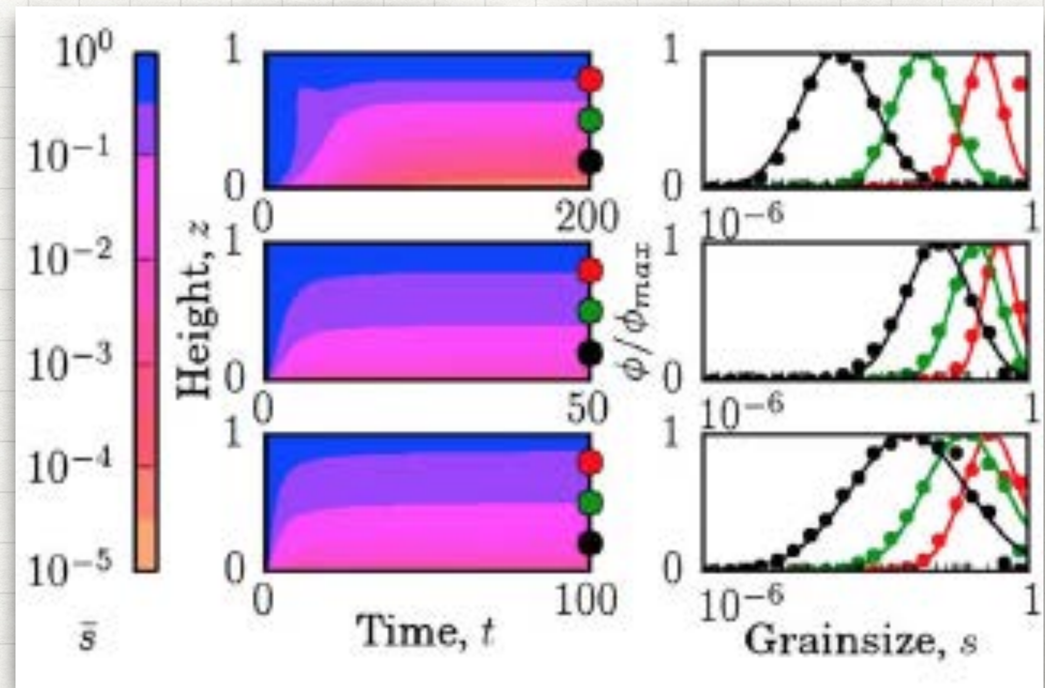


CATACLASTIC GRANULAR FLOWS

- STOCHASTIC HETERARCHY







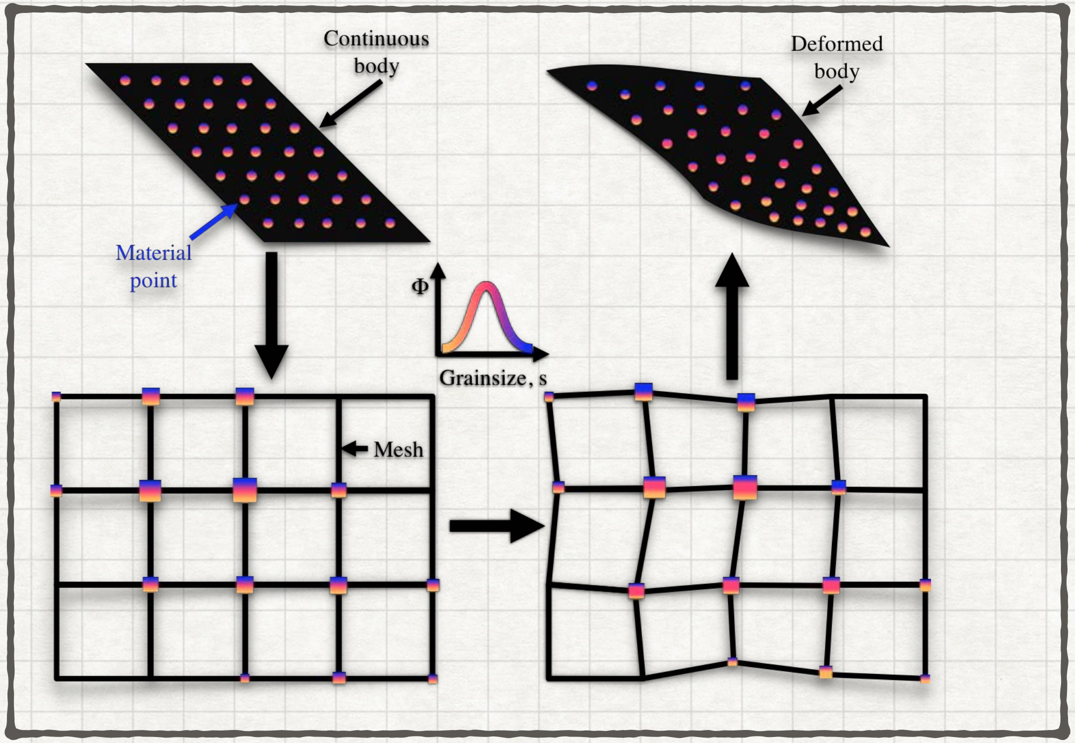
model results

Marks-Einav, GRL, 2015

LONG-TERM PROJECT

- HETERARCHICAL CONTINUUM SOLVER

MPM with
Benjy Marks



Segregation +
Mixing +
Energetics …

+
Comminution…

$$\rho^{t+1}(\psi,\bar{\psi}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d\phi\, d\bar{\phi}\, \rho^{t}(\phi,\bar{\phi})\, \kappa^{g^*}(\phi-\psi,\bar{\phi}-\bar{\psi})$$

HETERARCHICAL CONCLUSIONS

- Alternative approach to multiscale hierarchal models;
- Stochastic laws homoganisable into equivalent continua;

MINERAL PROCESSING

- Agglomeration, screening, crushers…
- Grainshape dynamics…

GENERAL SCOPE

- Other engineering problems with no scale separation
- e.g., weather and environmental patterns