Structural analysis and design -
What we do, and what we could do

Presentation to the Data-centric Engineering Group

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Structural Engineering vs Mechanical Engineering

- Structural design checks limit states including:
  - Ultimate limit state
  - Serviceability limit state
- The ultimate limit state may be complete collapse, i.e.
  - The displacements are nonlinear
  - The material behaves nonlinearly
- Structural stability is affected by initial geometric imperfections, e.g.
  - Out-of-plumb
  - Out-of-straightness
Structural Design – What we do

- Structural design is probabilistically based
- Design criteria are in terms of probability of failure

Design action \( S^*(x,t) = S[D(x), L(x,t), S(x,t), W(x,t), E(x,t), ...] \)

Capacity \( R = R(x,t) = R[M(x,t), F(x)] \)

- \( S^* \) and \( R \) are random variables
- Generally: \( S^* = S^*(x) \) and \( R = R(x) \)
Structural Design – Actions (loads)

- Loads are expressed in terms of probability of exceedance
- AS/NZS 1170 Structural design actions. Part 0: General principles

<table>
<thead>
<tr>
<th>Importance level</th>
<th>Wind</th>
<th>Snow</th>
<th>Earthquake</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1/100</td>
<td>1/50</td>
<td>1/100</td>
</tr>
<tr>
<td>2</td>
<td>1/500</td>
<td>1/150</td>
<td>1/500</td>
</tr>
<tr>
<td>3</td>
<td>1/1000</td>
<td>1/250</td>
<td>1/1000</td>
</tr>
<tr>
<td>4</td>
<td>1/2000</td>
<td>1/500</td>
<td>1/2500</td>
</tr>
</tbody>
</table>

- Statistics are available for loads (mean, CoV, and distribution)
- E.g. D - Normal, L - Gamma distribution, W - Extreme Type I distribution
Structural Design – Capacity (Strength)

– The strength of a structure depends on the material (M) and geometric (F) properties

– M: Provides relationship between stress and strain, typically (E, f_y, f_u, ...) and a stress-strain curve + yield surface for combined stress, flow rules, etc.
  – Steel, aluminium, etc (metals): M=M(x)
  – Concrete, timber, etc: M=M(x,t)

– F: Tolerance in fabrication and erection, typically (t, b, imperfections)

– Statistics are available for common M and F, e.g. f_y – Lognormal; E – normal; t,b – Lognormal etc.
Structural Design – design criteria

– Probability of failure

\[ P_f = P_r[R - S^* \leq 0] = P_f[g(R, S^*) \leq 0] = \int \cdots \int \frac{f_x(x)}{g(x) \leq 0} dx \]

– \( X \) – vector of basic random variables, e.g. \( E, f_y, D, L \), etc.

– \( f_x(x) \) – joint probability density function (PDF) of basic variable

– \( g \) – failure function, \( g(.) \leq 0 \) implies failure, e.g. Dead and Live load combination:

\[ g = R - D - L \]

– Reliability index (\( \beta \))

\[ \beta = \Phi^{-1}(1 - P_f) \]

– Design criterion:

\[ P_f \leq P_f^0 \quad \text{or} \quad \beta \geq \beta_0 \]
Structural Design – design criteria con’t

– Practical design uses nominal values of random variable, e.g. $E_n, f_y, D_n, L_n$, etc

– Design check:

$$\varphi R_n \geq \sum \gamma_i S^*_{ni}$$

– $\sum \gamma_i S^*_{ni}$ is the load combination,
  – Dead and live: $\sum \gamma_i S^*_{ni} = 1.2D_n + 1.5L_n$
  – Dead, live and wind: $\sum \gamma_i S^*_{ni} = 1.2D_n + \psi_c L_n + W_n$
  – Dead live and earthquake: $\sum \gamma_i S^*_{ni} = D_n + \psi_c L_n + E_n$

– $\varphi$ is the resistance factor

– Reliability calibration: Determine $\varphi$ so that $\beta \geq \beta_0$ is satisfied
  – Member level
  – System level
Structural Design – reliability calibration

- Member-based design
  - Build model, apply loads, run analysis (elastic) \( \rightarrow M, N, V \ldots \)
  - Design check (AS4100), e.g. simple beam: \( \phi M_p \geq M \)
  - Must be satisfied for all members and connections

- System-based design
  - Build model, apply loads \( (D, L, W \ldots) \) and introduce a load increment factor \( (\lambda) \), run fully nonlinear analysis (mimic actual behaviour)
  - Design check: \( \phi_s \lambda_u \geq 1 \)
Systems reliability calibration

Reliability analysis framework

<table>
<thead>
<tr>
<th>Member Set</th>
<th>$\lambda_{un}$</th>
<th>$\varphi_s = \frac{1}{\lambda_{un}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.58</td>
<td>0.63</td>
</tr>
<tr>
<td>2</td>
<td>1.46</td>
<td>0.68</td>
</tr>
<tr>
<td>5</td>
<td>1.04</td>
<td>0.96</td>
</tr>
</tbody>
</table>

ii) Select cross-sections $\Rightarrow$ different failure modes

Specify loading, e.g. gravity,
and load ratios, e.g. $L_n/D_n$

iii) Run Monte-Carlo simulations
on random frames $\Rightarrow R$

Random variables:

Material: $f_y$, $E$ and $\sigma_r$

Geometry: $t$, $b$, geom. imperfections

Modelling uncertainty

iv) Reliability analysis (e.g. FORM)

$\beta = \Phi^{-1}(P_f)$

$\varphi_s$ vs $\beta$

v) Plot $\beta$ vs $\varphi_s$ curves

Repeat for other load ratios

Determine $\varphi_s$ for given $\beta$

Repeat for other member sets

Repeat for other frames
Development of system-based design framework

- DP11 (Rasmussen, Zhang, Ellingwood): Connections assumed not to fail
- DP16 (Rasmussen, Zhang, da Silva): Connection models included in calibration
- DP19 (Rasmussen, Zhang, Khezri, Deierlein): FE Modelling of connections including fracture – all limit states checked

- Finer discretisation, nDOF becoming large
- CPU time is becoming an issue
Structures – Data-intensive applications

– Wind pressure on buildings

– Tall buildings are conventionally designed using expensive wind tunnel tests on scaled models
– Wind tunnel tests predict the pressure distribution on the building
– Simple linear interpolation prove inaccurate in highly nonlinear regions
– Machine learning techniques have proven effective to undertake regression and classification tasks in wind-related applications

Constitutive (material) modelling

- Conventionally approach: Condensed experimental data into deterministic laws that are coded in FE software
- Alternative: Use ML to extract stress-strain relationships from experimental or prior data, and implement routines in FE simulations

\[
\begin{align*}
\sigma_2 + \alpha_1 \sigma_1 - \gamma T \left( \frac{\sigma_1}{X^C} + 1 \right) &= 0 \\
\frac{\sigma_1}{X^C} + 1 &= 0 \\
\frac{\sigma_1}{X^C} &= \frac{1 + \beta_x \left( \sigma_2 / \sigma_1 \right)}{(1 + \sigma_2 / \sigma_1)^2} \\
\sigma_2 &= \frac{1 + \beta_y \left( \sigma_1 / \sigma_2 \right)}{(1 + \sigma_1 / \sigma_2)^2} \\
\frac{\sigma_2}{Y^C} + 1 &= 0 \\
\sigma_1 + \alpha_2 \sigma_2 - X_T \left( \frac{\sigma_2}{Y^C} + 1 \right) &= 0
\end{align*}
\]

Note: \( \alpha_1 \) and \( \alpha_2 \) are slope of the dashed lines
Structures – Data-intensive applications con’t

Gaussian process regression-based constitutive models

Gaussian process regression (GPR) based Constitutive Modelling

Advantages

1. Both underlying relation and uncertainty of data could be captured
2. Underlying relation is expressed as a stochastic function transparently
3. No assumption on the model expression is required
4. Suitable for all materials

Data Driven

Experimental data

Gaussian process regression algorithm

Data driven constitutive model
Structures – Data-intensive applications con’t

Gaussian process regression-based constitutive models, cont

Gaussian Process (GP) define a distribution over a stochastic function.

\[ f \sim \mathcal{GP}(m(x), k(x, x')) \]

Mean covariance Function values of \( n \) positions, \( \{f(x_1^*), f(x_2^*), \ldots, f(x_i^*), \ldots, f(x_n^*)\} \) or \( f(x^*) \), comply the multivariable Gaussian distribution

\[ f(x^*) \sim \mathcal{N}(m(x^*), K^{**}) \]

\[
K^{**} =
\begin{bmatrix}
  k(x_1^*, x_1^*) & k(x_1^*, x_2^*) & \ldots & k(x_1^*, x_n^*) \\
  k(x_2^*, x_1^*) & k(x_2^*, x_2^*) & \ldots & k(x_2^*, x_n^*) \\
  \vdots & \vdots & \ddots & \vdots \\
  k(x_n^*, x_1^*) & k(x_n^*, x_2^*) & \ldots & k(x_n^*, x_n^*)
\end{bmatrix}
\]

Three samples of the function distributed as the GP with:

- mean function \( m(x) = 0.25x^2 \) and
- covariance function \( k(x, x') = \exp\left[-\frac{(x-x')^2}{2}\right] \)
Structures – Data-intensive applications con’t

– Gaussian process regression-based constitutive models con’t

Loads P1 and P1 – $N(15, 1.5)$ kN/m
Load W – Gamma (20, 7.4) kN

• Data driven stochastic structural analysis is performed using Monte Carlo (MC) simulations (N=3000, Abaqus used)

• Both load uncertainty and material uncertainty are considered in this example

• The critical value of the deflection at point G is $R_c = H/500$ (20 mm)
### Gaussian process regression-based constitutive models con’t

<table>
<thead>
<tr>
<th>COV</th>
<th>Dataset size</th>
<th>Expectation (mm)</th>
<th>Standard deviation (mm)</th>
<th>Probability of failure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>True</td>
<td>GPR</td>
<td>True</td>
</tr>
<tr>
<td>0.05</td>
<td>200</td>
<td></td>
<td>13.51</td>
<td>5.64</td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>13.57</td>
<td>13.54</td>
<td>5.67</td>
</tr>
<tr>
<td></td>
<td>800</td>
<td>13.55</td>
<td>5.65</td>
<td>5.64</td>
</tr>
<tr>
<td>0.10</td>
<td>200</td>
<td></td>
<td>13.79</td>
<td>6.12</td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>13.81</td>
<td>13.84</td>
<td>6.02</td>
</tr>
<tr>
<td></td>
<td>800</td>
<td>13.83</td>
<td>6.07</td>
<td>6.08</td>
</tr>
<tr>
<td>0.15</td>
<td>200</td>
<td></td>
<td>14.72</td>
<td>6.17</td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>13.94</td>
<td>14.07</td>
<td>6.26</td>
</tr>
<tr>
<td></td>
<td>800</td>
<td>14.03</td>
<td>6.22</td>
<td>6.21</td>
</tr>
</tbody>
</table>

- A larger dataset size is required to obtain a good estimation of the expected deflection for data with high uncertainty level.
- The probability of failure increases with the uncertainty level increasing.
- Without considering the material uncertainty accurately, the probability of failure will be underestimated.
- The probability of failure predicted by using the GPR model is conservative and will converge to the reference value with the number of data points increasing.
Structures – Data-intensive applications con’t

Deep learning-based method on seismic fragility analysis of bridges considering aging effects

- Seismic fragility: Conditional probability providing the likelihood of a structure (or component) exceeding a predefined level of damage for a given ground motion intensity

- Conventional fragility analysis method requires a series of nonlinear time history analysis (computationally expensive)

- Conventional fragility analysis method is not practical for:
  - time-dependent seismic fragility analysis for deteriorating facilities considering aging effects
  - seismic assessment for a transportation network with many bridges

- For highly repetitive analyses, deep learning models can be good surrogates with high accuracy and efficiency. The mechanism of generating fragility curves using deep learning can be simply regarded as a decision-making process, which compares demand with capacity to classify whether the bridge exceeds the limit state or not.
Deep learning-based method on seismic fragility analysis of bridges considering aging effects, con’t

- Conventional fragility analysis process

<table>
<thead>
<tr>
<th>Suite of Ground Motions</th>
<th>Bridge Model Simulations</th>
<th>Bridge Component Responses</th>
<th>Probabilistic Seismic Demand Models</th>
<th>Capacity</th>
<th>Fragility Models</th>
</tr>
</thead>
</table>

1st sample

$f_{C1}, f_{yS1}, \ldots$

kth sample

$f_{Ck}, f_{ySk}, \ldots$

Nth sample

$f_{CN}, f_{ySN}, \ldots$

Component 1

$H(r)$

Limit States

$S_{w \ (g)}$

Component M

$H(r)$

Limit States

$S_{w \ (g)}$
Structures – Data-intensive applications con’t

Deep learning-based method on seismic fragility analysis of bridges considering aging effects, con’t

– Alternative: For highly repetitive analyses, deep learning models can be good surrogates with high accuracy and efficiency
– Generating fragility curves using deep learning can be regarded as a decision-making process, which compares demand with capacity to determine whether a limit state is exceeded or not
– Problem can be transformed into a binary classification problem

Model Selection, Training and Test
Deep learning-based method on seismic fragility analysis of bridges considering aging effects, con’t

- Results to date suggest the use of ML proves sufficient accuracy.

Fragility curves developed by the DNN-based method and the traditional NLTHA method.
Structures – What we could do

Error identification in structural design

– Structural members and connections in steel framework tend to be standardised
– Each requires a calculation of strength and deformation response to a structural design code
– Thousands of strength calculations are available in design offices and could be used to train predictive models using ML
– Useful for design or error identification in design
Structures – What we could do

Optimised structural design equations

1. Large experimental data sets are available for common types of structural members
2. Could be used to train predictive algorithms to optimise efficiency

1. Input parameters: Select input variables that have the most significant effects on the output variable(s), (h, b, d, t, L, f_y, ...)
2. Choose ML algorithm to generate reasonably simple predictive models
3. Divide datasets into two subsets (training/testing) model to generate and verify predictive model.
Structures – What we could do

Reformulate problems in terms of Bayesian statistics

– Where would this make sense?
  – In problems requiring probabilistic assessment, e.g. reliability analysis
  – In problems with a high degree of uncertainty, e.g. constitutive modelling of materials with high degrees of variability, (timber, 3D printed materials, …)
  

– In time-dependent problems where data is collected that can inform the model, e.g. structural health monitoring of significant infrastructure (bridges, tunnels, tall buildings, …)

– In problems where the loading model is associated with significant reliability, e.g. design of wind turbine towers

– There is a field of research on statistical finite element analysis
Structures – What we could do

Reformulate problems in terms of Bayesian statistics, con’t
– What governing equations lend themselves to this?
  – Dynamic structural analysis (vibrations, earthquake,...)
    \[ m \frac{\partial^2 u(x,t)}{\partial t^2} + c \frac{\partial u(x,t)}{\partial t} + ku(x,t) = f(x,t) \]
  – Transport problems (advection-diffusion-reaction), e.g. carbonation, chloride ingress, hydration in concrete
    \[ \frac{\partial \phi}{\partial t} + \frac{\partial}{\partial x_i} \left( \beta_i \phi - D_{ij} \frac{\partial \phi}{\partial x_j} \right) + \lambda \phi = Q \]


– Fluid dynamics in wind and water applications, and fluid-structure interaction applications
  \[ \frac{\partial u}{\partial t} + (u \cdot \nabla)u - \nu \nabla^2 u = g - \nabla p \]