Structural analysis and design - What we do, and what we could do

Presentation to the Data-centric Engineering Group

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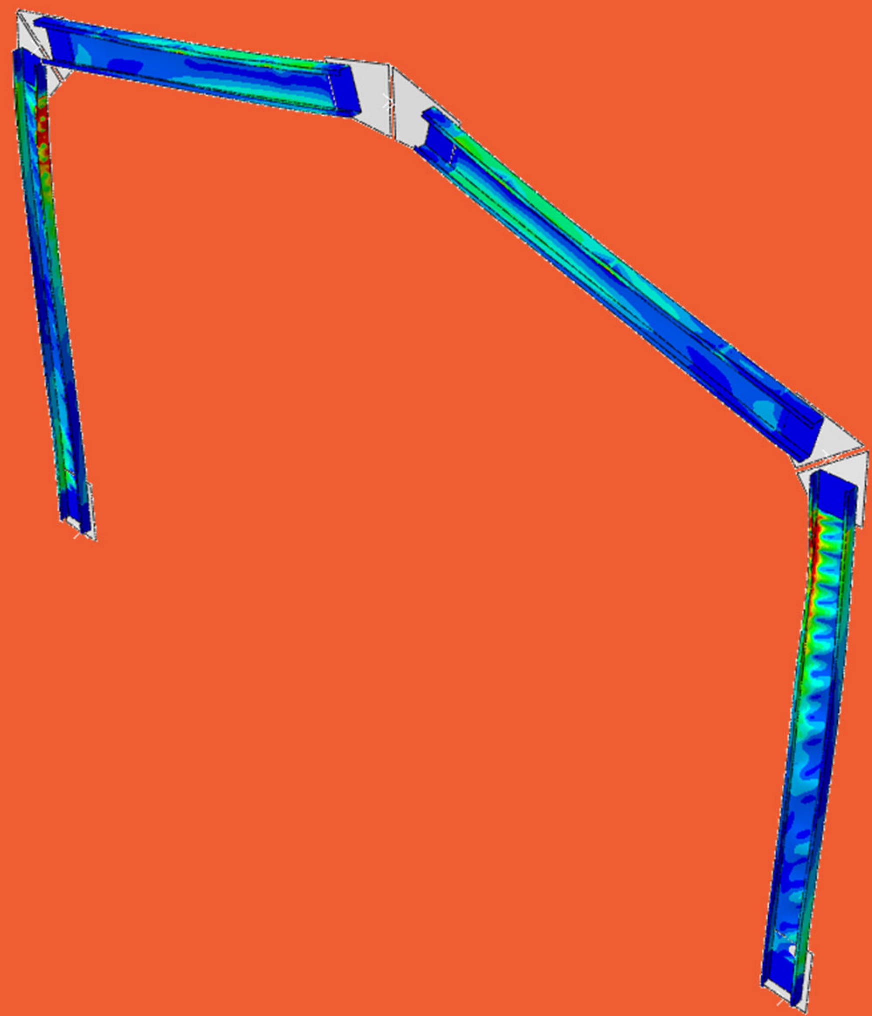
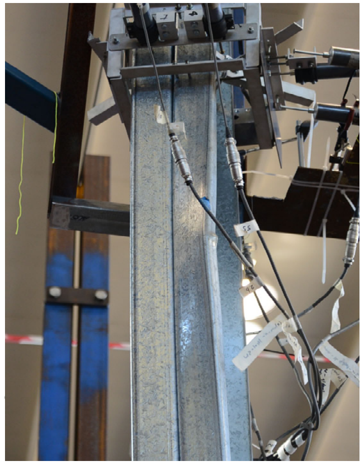


Structural Engineering *vs* Mechanical Engineering

- Structural design checks limit states including:
  - Ultimate limit state
  - Serviceability limit state
- The ultimate limit state may be complete collapse, i.e.
  - The displacements are nonlinear
  - The material behaves nonlinearly
- Structural stability is affected by initial geometric imperfections, e.g.
  - Out-of-plumb
  - Out-of-straightness



Structural Design – What we do

– Structural design is probabilistically based
– Design criteria are in terms of probability of failure

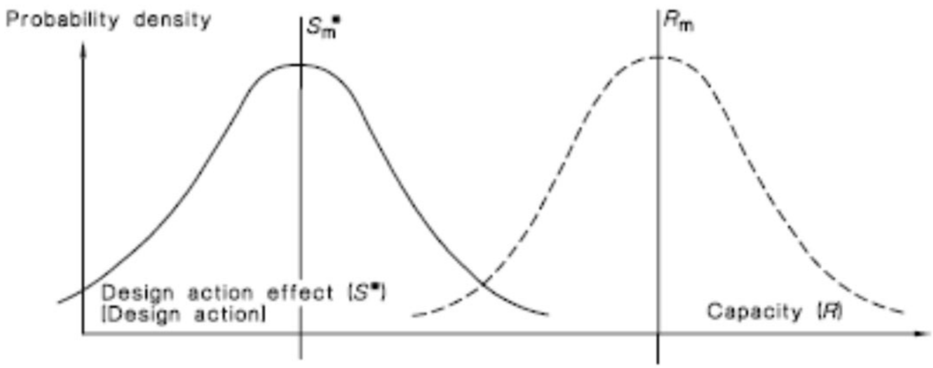


FIGURE C1.6.2.2(1) RANDOMNESS S^* AND R

– Design action $S^* = S^*(\mathbf{x},t) = S[D(\mathbf{x}), L(\mathbf{x},t), S(\mathbf{x},t), W(\mathbf{x},t), E(\mathbf{x},t), \ldots]$
– Capacity $R = R(\mathbf{x},t) = R[M(\mathbf{x},t), F(\mathbf{x})]$
– S^* and R are random variables
– Generally: $S^* = S^*(\mathbf{x})$ and $R = R(\mathbf{x})$

Structural Design – Actions (loads)

– Loads are expressed in terms of probability of exceedance
– AS/NZS 1170 Structural design actions. Part 0: General principles

TABLE 3.2

REFERENCE PROBABILITY OF EXCEEDANCE

Importance level	Annual probability of the design event for safety		
	Wind	Snow	Earthquake
1	1/100	1/50	1/100
2	1/500	1/150	1/500
3	1/1000	1/250	1/1000
4	1/2000	1/500	1/2500

– Statistics are available for loads (mean, CoV, and distribution)
– E.g. D - Normal, L - Gamma distribution, W - Extreme Type I distribution

Structural Design – Capacity (Strength)

– The strength of a structure depends on the material (M) and geometric (F) properties

– M: Provides relationship between stress and strain, typically (E, f_y, f_u, …) and a stress-strain curve + yield surface for combined stress, flow rules, etc.

  – Steel, aluminium, etc (metals): $M=M(\mathbf{x})$

  – Concrete, timber, etc: $M=M(\mathbf{x},t)$

– F: Tolerance in fabrication and erection, typically (t, b, imperfections)

– Statistics are available for common M and F, e.g. fy – Lognormal; E – normal; t,b – Lognormal etc.

Structural Design – design criteria

– Probability of failure

$$P_f = \mathrm{P}_r[R - S^* \leq 0] = \mathrm{P}_f[g(R, S^*) \leq 0] = \int \dots \int_{g(\boldsymbol{X}) \leq 0} f_{\boldsymbol{x}}(\boldsymbol{x}) d\boldsymbol{x}$$

– **X** – vector of basic random variables, e.g. E, f_y, D, L, etc.

– $f_{\mathbf{x}}(\mathbf{x})$ – joint probability density function (PDF) of basic variable

– g – failure function, g(.) ≤ 0 implies failure, e.g. Dead and Live load combination:

$$g = R - D - L$$

– Reliability index (β)

$$\beta = \Phi^{-1}(1 - P_f)$$

– Design criterion:

$$P_f \leq P_f^0 \quad \text{or} \quad \boxed{\beta \geq \beta_0}$$

Structural Design – design criteria con't

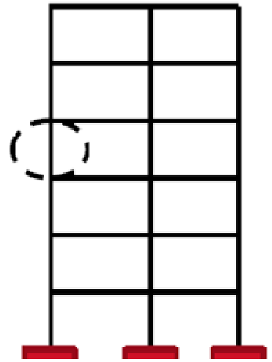
- Practical design uses nominal values of random variable, e.g. E_n, f_y, D_n, L_n, etc
- Design check:

$$\varphi R_n \geq \sum \gamma_i S_{ni}^*$$

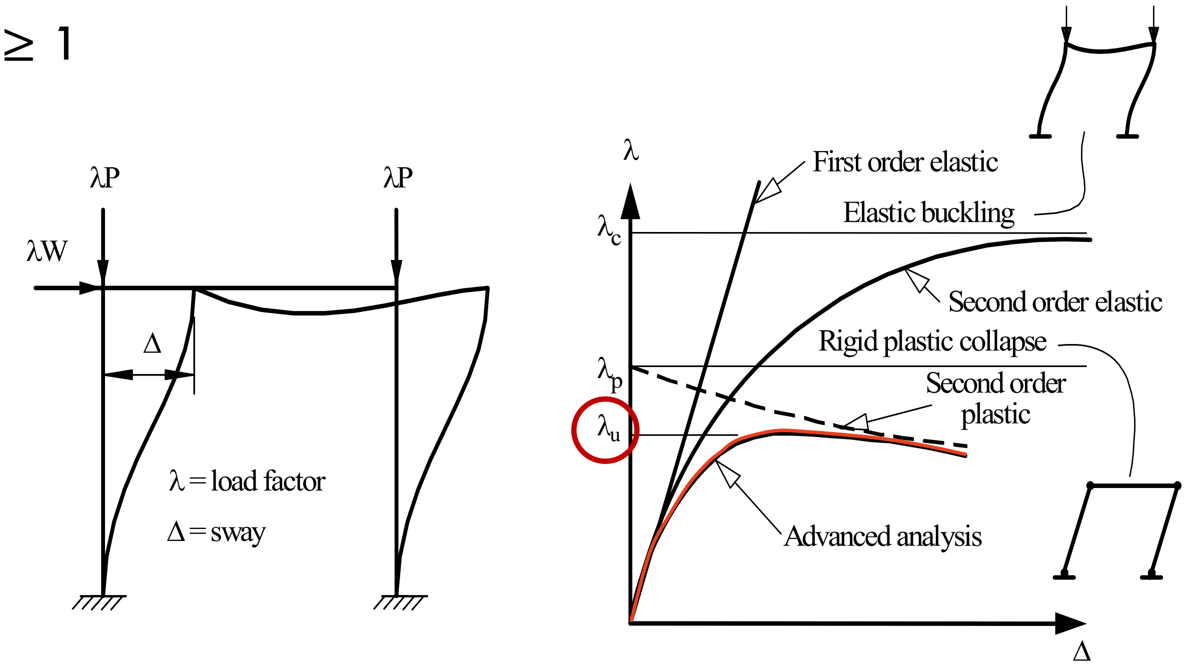
- $\sum\gamma_i S^*_{ni}$ is the load combination,
  - Dead and live: $\sum\gamma_i S^*_{ni} = 1.2D_n+1.5L_n$
  - Dead, live and wind: $\sum\gamma_i S^*_{ni} = 1.2D_n+\psi_c L_n+W_n$
  - Dead live and earthquake: $\sum\gamma_i S^*_{ni} = D_n+\psi_c L_n+E_n$
- φ is the resistance factor

- Reliability calibration: Determine φ so that $\beta \geq \beta_0$ is satisfied
  - Member level
  - System level

Structural Design – reliability calibration



- Member-based design
  - Build model, apply loads, run analysis (elastic) → M, N, V …
  - Design check (AS4100), e.g. simple beam: $\varphi M_p \geq M$
  - Must be satisfied for all members and connections
- System-based design
  - Build model, apply loads (D, L, W…) and introduce a load increment factor (λ), run fully nonlinear analysis (mimic actual behaviour)
  - Design check: $\varphi_s \lambda_u \geq 1$

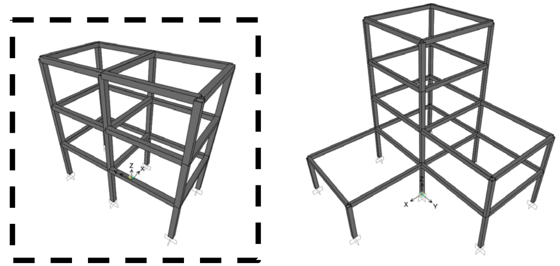


(a) Rigid jointed sway frame

(b) Load deflection responces

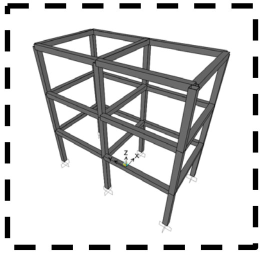
Systems reliability calibration

Reliability analysis framework



i) Select sample frames

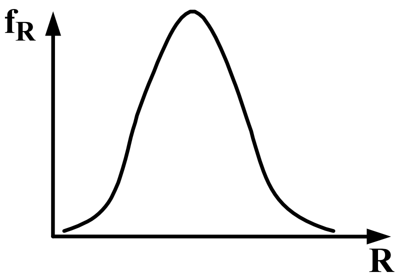
Member Set	λ_{un}	$\varphi_s = 1/\lambda_{un}$
1	1.58	0.63
2	1.46	0.68
⋮	⋮	⋮
5	1.04	0.96



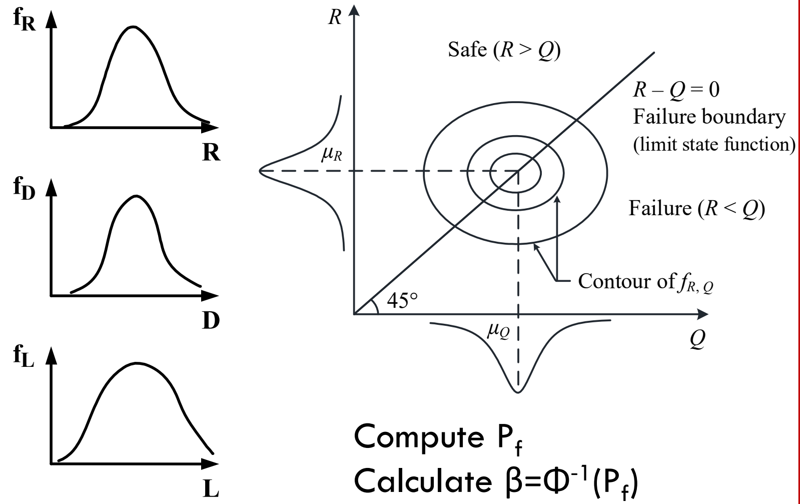
ii) Select cross-sections ⇒ different failure modes
Specify loading, e.g. gravity,
and load ratios, e.g. L_n/D_n

iii) Run Monte-Carlo simulations
on random frames ⇒ R

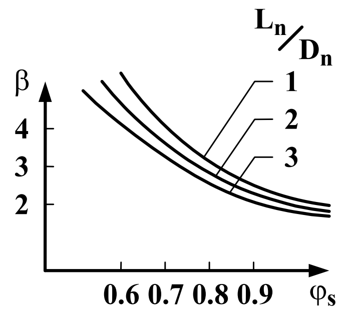
Random variables:
Material: f_y , E and σ_r
Geometry: t, b, geom. imperfections
Modelling uncertainty



iv) Reliability analysis (e.g. FORM)



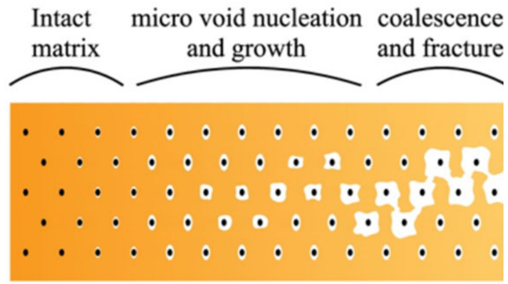
Compute P_f
Calculate $\beta=\Phi^{-1}(P_f)$

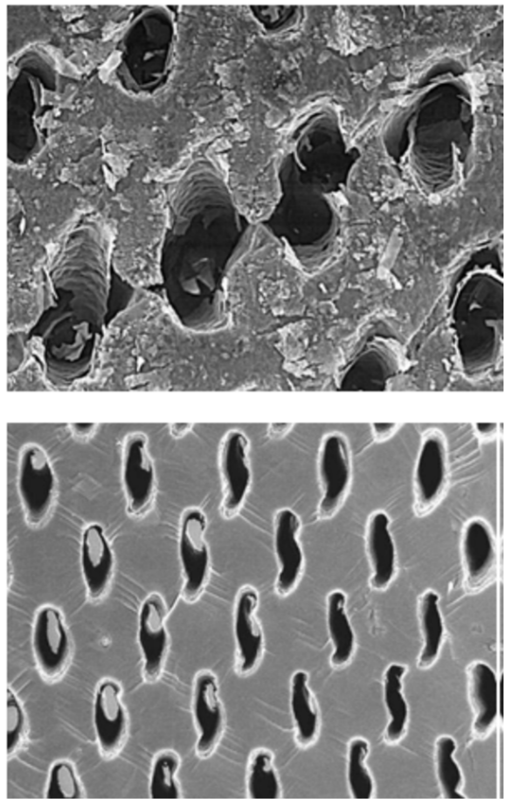


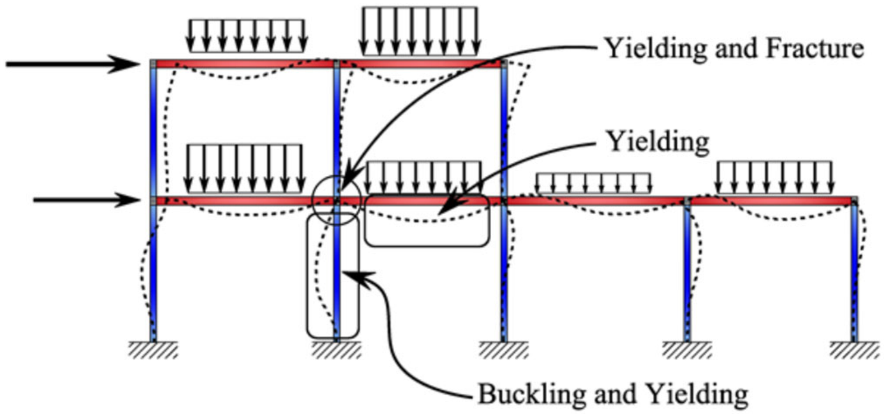
v) Plot β vs φ_s curves
Repeat for other load ratios
Determine φ_s for given β
Repeat for other member sets
Repeat for other frames

Development of system-based design framework

– DP11 (Rasmussen, Zhang, Ellingwood): Connections assumed not to fail
– DP16 (Rasmussen, Zhang, da Silva): Connection models included in calibration
– DP19 (Rasmussen, Zhang, Khezri, Deierlein): FE Modelling of connections including fracture – all limit states checked



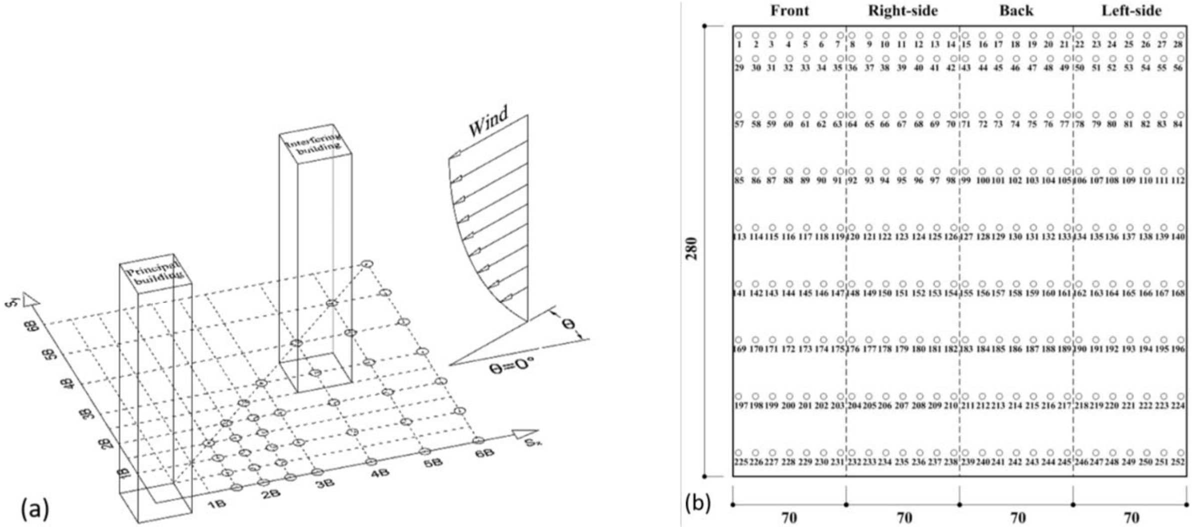




– Finer discretisation, nDOF becoming large
– CPU time is becoming an issue

Structures – Data-intensive applications

– Wind pressure on buildings



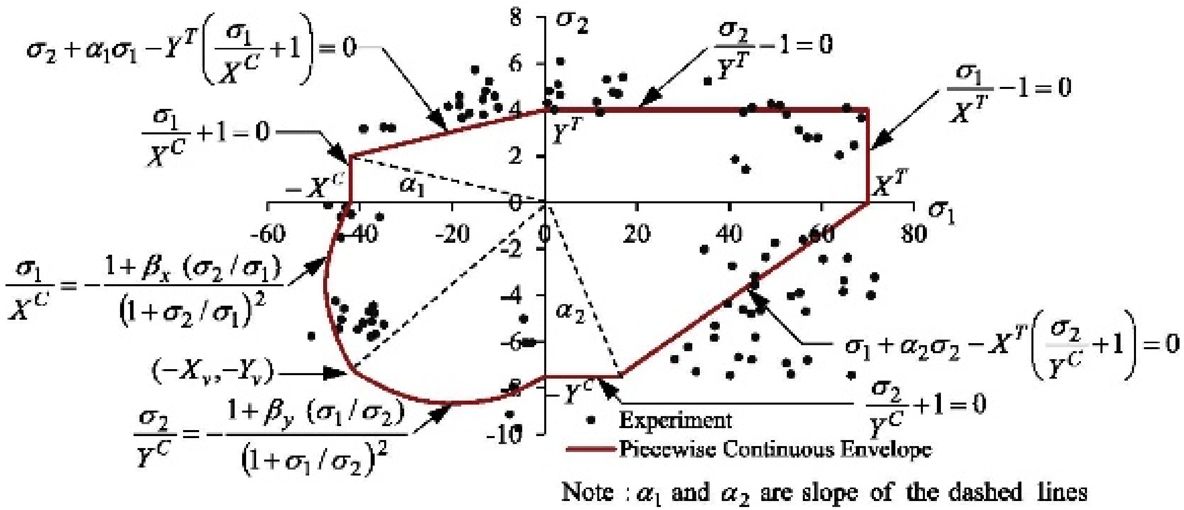
– Tall buildings are conventionally designed using expensive wind tunnel tests on scaled models
– Wind tunnel tests predict the pressure distribution on the building
– Simple linear interpolation prove inaccurate in highly nonlinear regions
– Machine learning techniques have proven effective to undertake regression and classification tasks in wind-related applications

Hu, G., Liu, L., Tao, D., Song, J., Kwok, K.C.S., Investigation of wind pressures on tall building under interference effects using machine learning techniques. CoRR abs/1908.07307 (2019)

Structures – Data-intensive applications con't

Constitutive (material) modelling

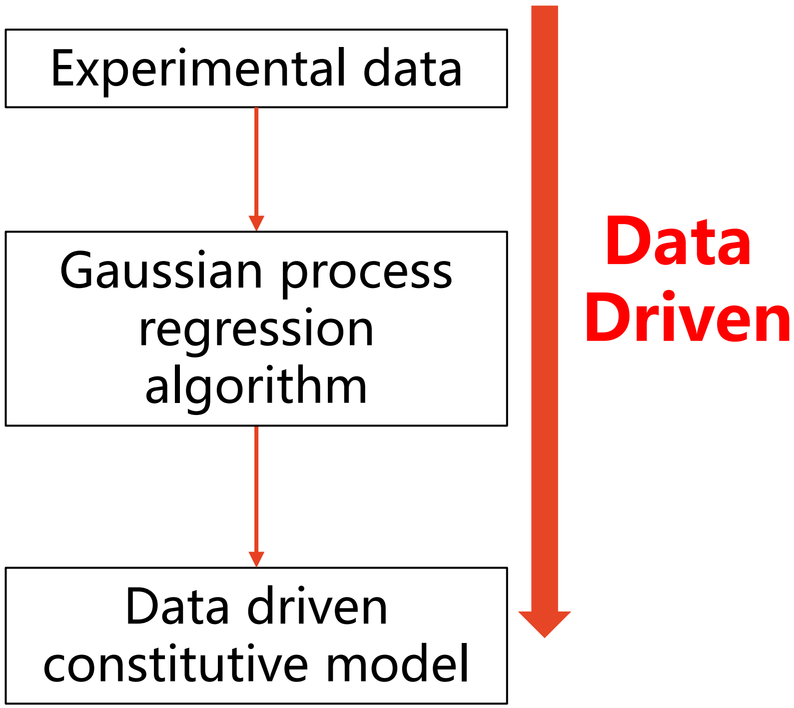
- Conventionally approach: Condensed experimental data into deterministic laws that are coded in FE software
- Alternative: Use ML to extract stress-strain relationships from experimental or prior data, and implement routines in FE simulations



Structures – Data-intensive applications con't

Gaussian process regression-based constitutive models

Gaussian process regression (GPR) based Constitutive Modelling



Advantages

1. Both underlying relation and uncertainty of data could be captured
2. Underlying relation is expressed as a stochastic function transparently
3. No assumption on the model expression is required
4. Suitable for all materials

Structures – Data-intensive applications con't

Gaussian process regression-based constitutive models, cont

Gaussian Process (GP) define a distribution over a stochastic function.

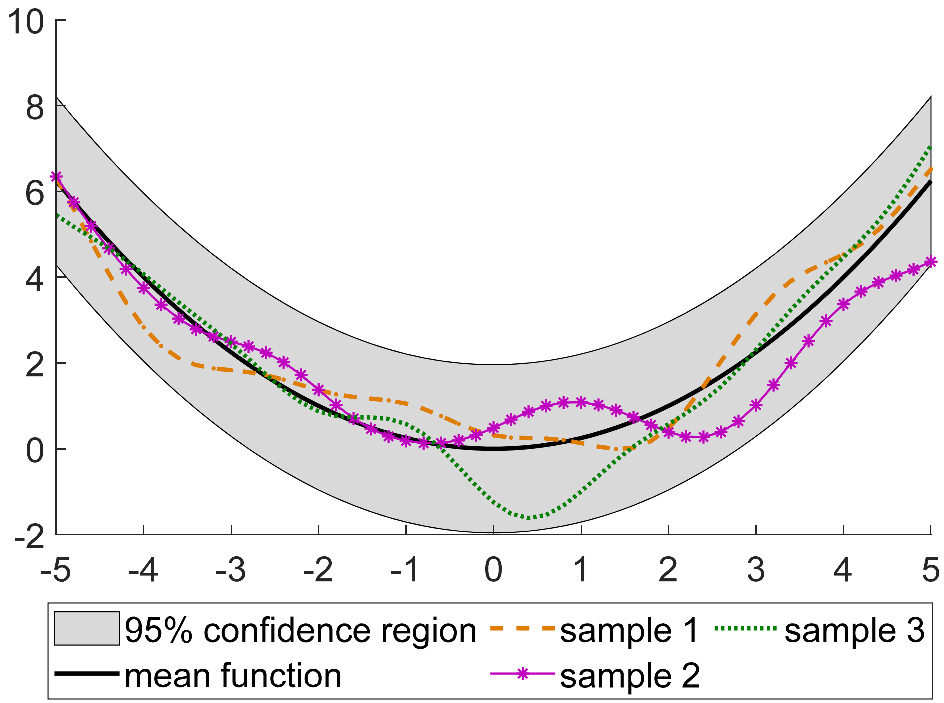
$$f \sim \mathcal{GP}(m(x), k(x, x'))$$

mean covariance

Function values of n positions, $\{f(x_1^*), f(x_2^*), \ldots, f(x_i^*) \ldots, f(x_n^*)\}$ or $f(\boldsymbol{x}^*)$, comply the multivariable Gaussian distribution

$$f(\boldsymbol{x}^*) \sim \mathcal{N}(\boldsymbol{m}(\boldsymbol{x}^*), \boldsymbol{K}_{**})$$

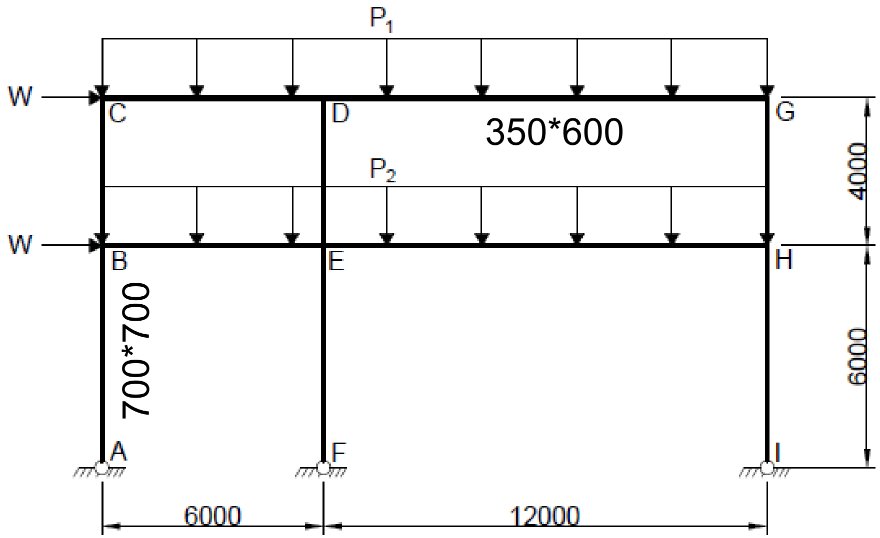
$$\boldsymbol{K}_{**} = \begin{bmatrix} k(x_1^*, x_1^*) & k(x_1^*, x_2^*) & \cdots & k(x_1^*, x_n^*) \\ k(x_2^*, x_1^*) & k(x_2^*, x_2^*) & \cdots & k(x_2^*, x_n^*) \\ \vdots & \vdots & \ddots & \vdots \\ k(x_n^*, x_1^*) & k(x_n^*, x_2^*) & \cdots & k(x_n^*, x_n^*) \end{bmatrix}$$



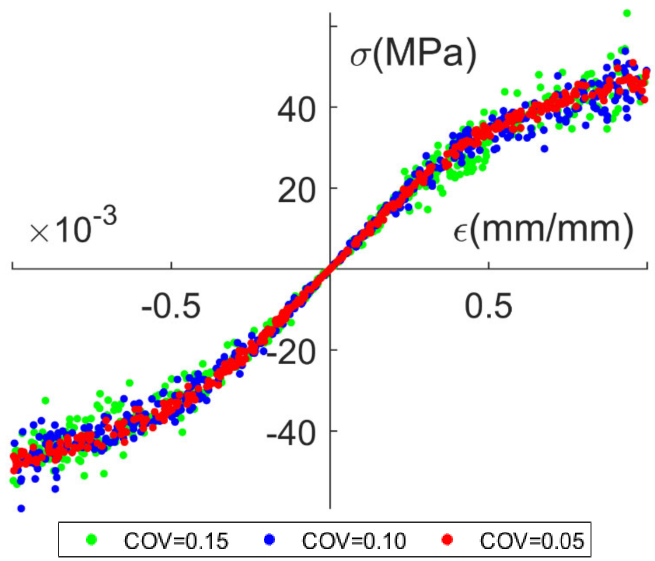
Three samples of the function distributed as the GP with:
mean function $m(x) = 0.25x^2$ and
covariance function $k(x, x') = \exp\left[\frac{-(x-x')^2}{2}\right]$

Structures – Data-intensive applications con't

– Gaussian process regression-based constitutive models con't



Loads P1 and P1 – $\mathcal{N}(15, 1.5)$ kN/m
Load W – *Gamma* (20, 7.4) kN



Stress-strain relation

- Data driven stochastic structural analysis is performed using Monte Carlo (MC) simulations (N=3000, Abaqus used)
- Both load uncertainty and material uncertainty are considered in this example
- The critical value of the deflection at point G is $R_c = \mathrm{H}/500$ (20 mm)

Chen, Shen and Zhang, *ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part A: Civil Engineering*, 2021

Structures – Data-intensive applications con't

Gaussian process regression-based constitutive models con't

COV	Dataset size	Expectation (mm)		Standard deviation (mm)		Probability of failure	
		True	GPR	True	GPR	True	GPR
0.05	200	13.57	13.51	5.67	5.64	12.8%	13.1%
	400		13.54		5.68		12.8%
	800		13.55		5.65		12.9%
0.10	200	13.81	13.79	6.02	6.12	14.6%	14.7%
	400		13.84		6.08		14.6%
	800		13.83		6.07		14.7%
0.15	200	13.94	14.72	6.26	6.17	15.5%	17.8%
	400		14.07		6.21		16.2%
	800		14.03		6.22		16.0%

- A larger dataset size is required to obtain a good estimation of the expected deflection for data with high uncertainty level
- The probability of failure increases with the uncertainty level increasing
- Without considering the material uncertainty accurately, the probability of failure will be underestimated
- The probability of failure predicted by using the GPR model is conservative and will converge to the reference value with the number of data points increasing

Structures – Data-intensive applications con't

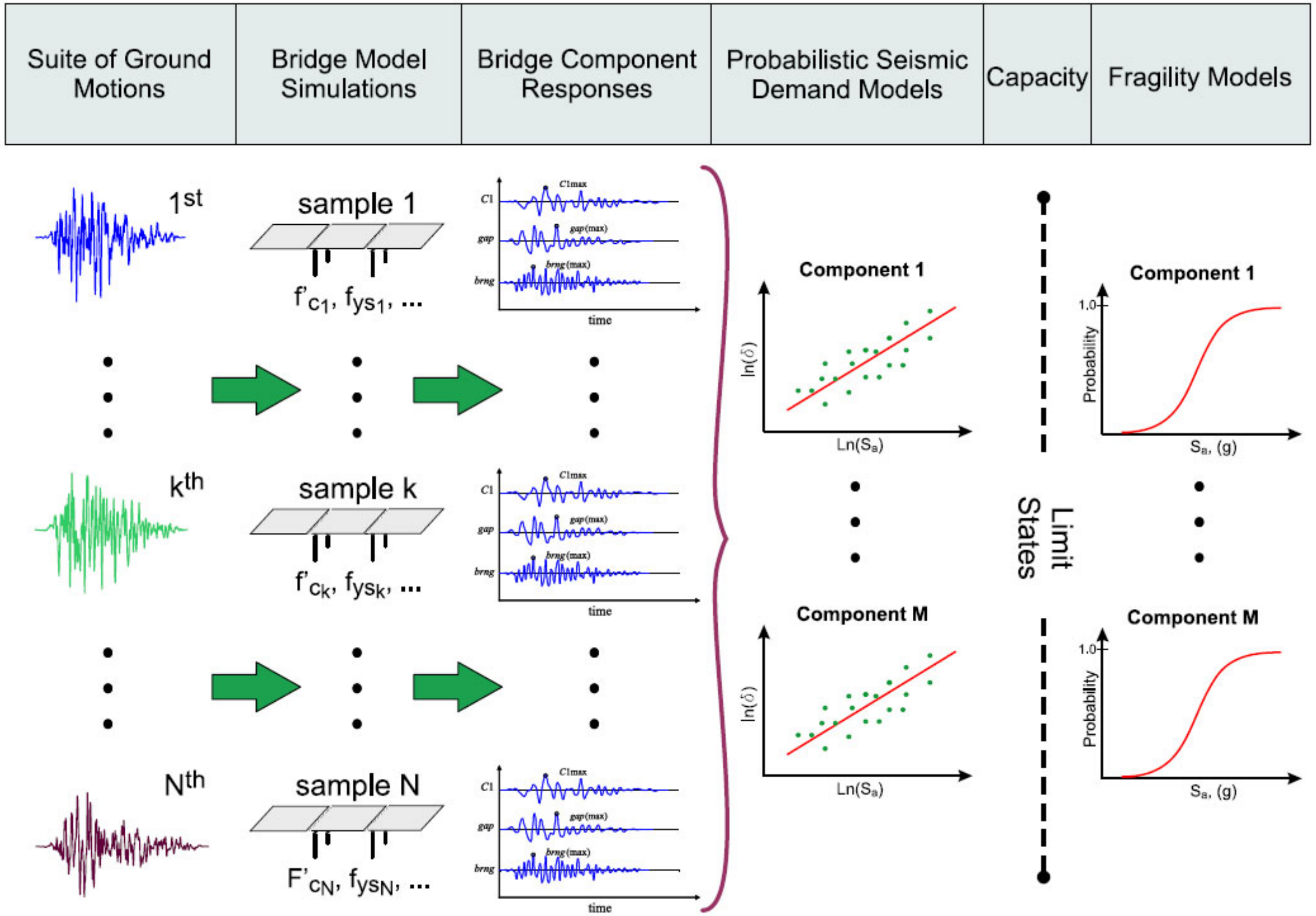
Deep learning-based method on seismic fragility analysis of bridges considering aging effects

- Seismic fragility: Conditional probability providing the likelihood of a structure (or component) exceeding a predefined level of damage for a given ground motion intensity
- Conventional fragility analysis method requires a series of nonlinear time history analysis (computationally expensive)
- Conventional fragility analysis method is not practical for:
  - time-dependent seismic fragility analysis for deteriorating facilities considering aging effects
  - seismic assessment for a transportation network with many bridges
- For highly repetitive analyses, deep learning models can be good surrogates with high accuracy and efficiency. The mechanism of generating fragility curves using deep learning can be simply regarded as a decision-making process, which compares demand with capacity to classify whether the bridge exceeds the limit state or not.

Structures – Data-intensive applications con't

Deep learning-based method on seismic fragility analysis of bridges considering aging effects, con't

– Conventional fragility analysis process

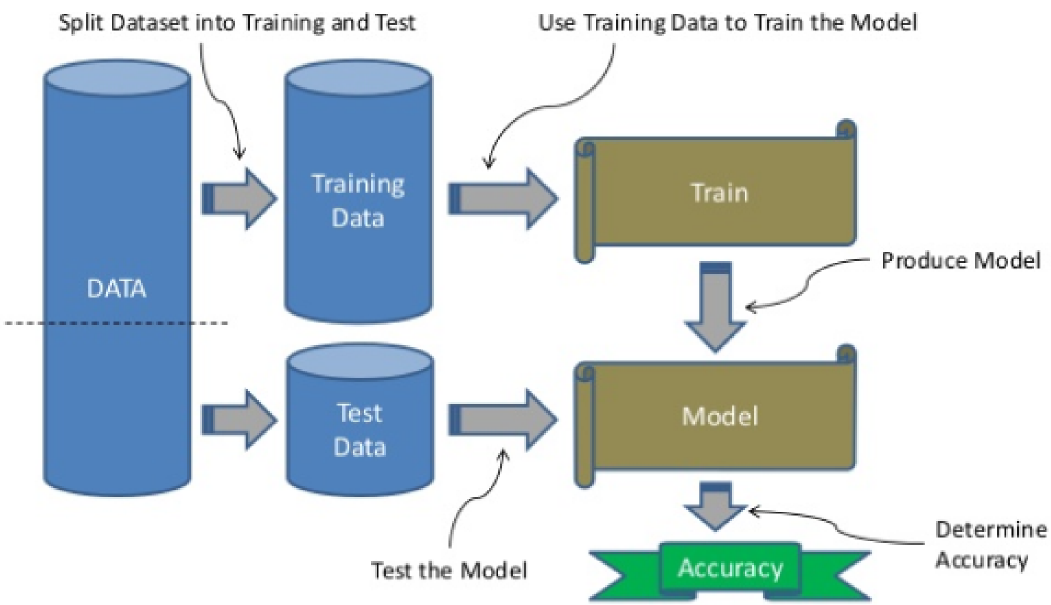


Structures – Data-intensive applications con't

Deep learning-based method on seismic fragility analysis of bridges considering aging effects, con't

– Alternative: For highly repetitive analyses, deep learning models can be good surrogates with high accuracy and efficiency
– Generating fragility curves using deep learning can be regarded as a decision-making process, which compares demand with capacity to determine whether a limit state is exceeded or not
– Problem can be transformed into a binary classification problem

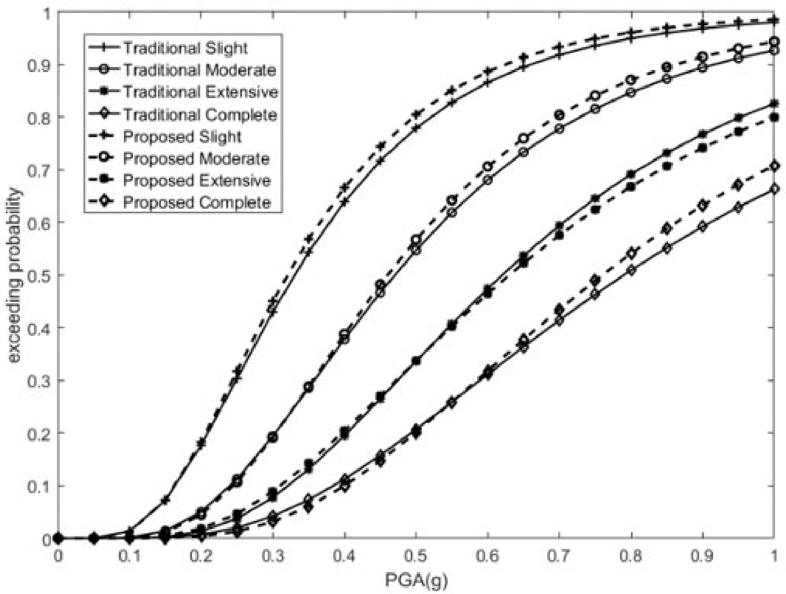
Model Selection, Training and Test



Structures – Data-intensive applications con't

Deep learning-based method on seismic fragility analysis of bridges considering aging effects, con't

– Results to date suggest the use of ML proves sufficient accuracy

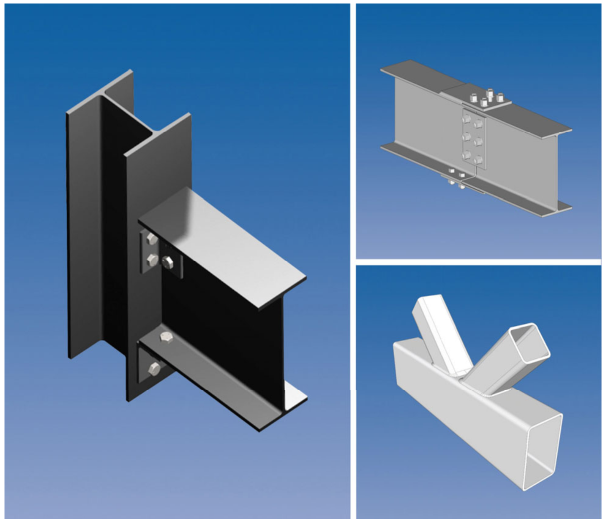


Fragility curves developed by the DNN-based method and the traditional NLTHA method.

Structures – What we could do

Error identification in structural design

- Structural members and connections in steel framework tend to be standardised
- Each requires a calculation of strength and deformation response to a structural design code
- Thousands of strength calculations are available in design offices and could be used to train predictive models using ML
- Useful for design or error identification in design



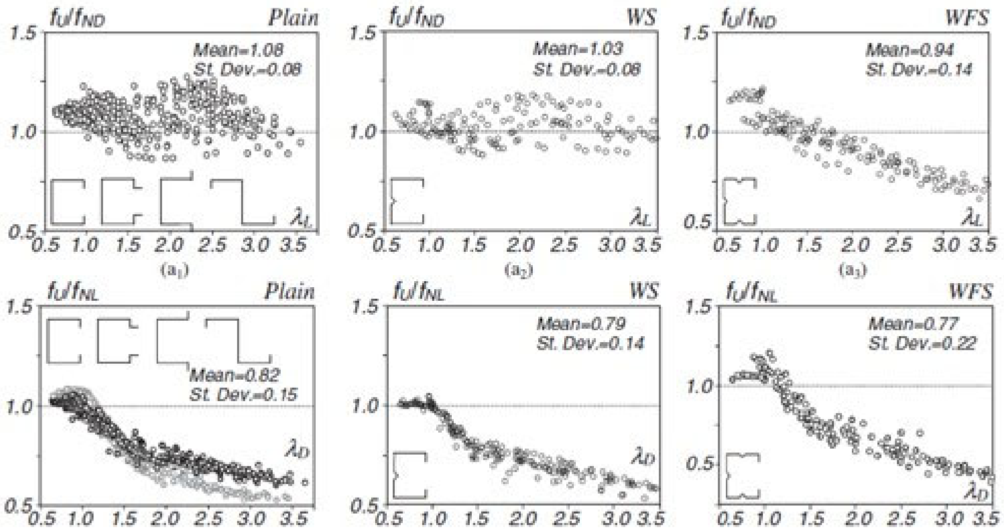
Structures – What we could do

Optimised structural design equations

– Large experimental data sets are available for common types of structural members

– Could be used to train predictive algorithms to optimise efficiency

1. Input parameters: Select input variables that have the most significant effects on the output variable(s), (h, b, d, t, L, f_y, …)
2. Choose ML algorithm to generate reasonably simple predictive models
3. Divide datasets is into two subsets (training/testing) model to generate and verify predictive model.



Structures – What we could do

Reformulate problems in terms of Bayesian statistics

- Where would this make sense?
  - In problems requiring probabilistic assessment, e.g. reliability analysis
  - In problems with a high degree of uncertainty, e.g. constitutive modelling of materials with high degrees of variability, (timber, 3D printed materials, …)

R. Ibanez, E. Abisset-Chavanne, Jose Vicente Aguado, David Gonzalez, Elias Cueto, Francisco Chinesta, A Manifold Learning Approach to Data-Driven Computational Elasticity and Inelasticity, Arch Computat Methods Eng (2018) 25:47–57

  - In time-dependent problems where data is collected that can inform the model, e.g. structural health monitoring of significant infrastructure (bridges, tunnels, tall buildings, …)
  - In problems where the loading model is associated with significant reliability, e.g. design of wind turbine towers
- There is a field of research on statistical finite element analysis

Structures – What we could do

Reformulate problems in terms of Bayesian statistics, con't

- What governing equations lend themselves to this?
  - Dynamic structural analysis (vibrations, earthquake,…)

$$\boldsymbol{m}\frac{\partial^2 \boldsymbol{u}(\boldsymbol{x},t)}{\partial t^2} + \boldsymbol{c}\frac{\partial \boldsymbol{u}(\boldsymbol{x},t)}{\partial t} + \boldsymbol{k}\boldsymbol{u}(\boldsymbol{x},t) = \boldsymbol{f}(\boldsymbol{x},t)$$

  - Transport problems (advection-diffusion-reaction), e.g. carbonation, chloride ingress, hydration in concrete

$$\frac{\partial \phi}{\partial t} + \frac{\partial}{\partial x_i}\left(\beta_i \phi - D_{ij}\frac{\partial \phi}{\partial x_j}\right) + \lambda\phi = Q$$

[1] M. Gharib, M. Khezri, S.J. Foster, Meshless and analytical solutions to the time-dependent advection-diffusion-reaction equation with variable coefficients and boundary conditions, Applied Mathematical Modelling 49 (2017) 220–242

[2] M. Gharib, M. Khezri, S.J. Foster, A. Castel, Application of the meshless generalised RKPM to the transient advection-diffusion-reaction equation, Computers and Structures, 193 (2017) 172–186

  - Fluid dynamics in wind and water applications, and fluid-structure interaction applications

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} - \nu\nabla^2\boldsymbol{u} = \boldsymbol{g} - \nabla p$$

  - …